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BOUNDARY LAYER ANALYSIS OF UPPER CONVECTED MAXWELL FLUID FLOW WITH VARIABLE THERMO-PHYSICAL PROPERTIES OVER A MELTING THERMALLY STRATIFIED SURFACE

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ABSTRACT
In this article, the boundary layer analysis which can be used to educate engineers and scientists on the flow of some fluids (i.e. glossy paints) in the industry is presented. The influence of melting heat transfer at the wall and thermal stratification at the freestream on Upper Convected Maxwell (UCM) fluid flow with heat transfer is considered. In order to accurately achieve the objective of this study, classical boundary condition of temperature is investigated and modified. The corresponding influences of thermal radiation and internal heat source across the horizontal space on viscosity and thermal conductivity of UCM are properly considered. The dynamic viscosity and thermal conductivity of UCM are temperature dependent. Classical temperature dependent viscosity and thermal conductivity models were modified to suit the case of both melting heat transfer and thermal stratification. The governing nonlinear partial differential equations describing the problem are reduced to a system of nonlinear ordinary differential equations using similarity transformations and solved numerically using the Runge-Kutta method along with shooting technique. The transverse velocity, longitudinal velocity and temperature of UCM are increasing functions of temperature dependent viscous and thermal conductivity parameters. This could increase the efficiency of some fluids flow in the industry. Effects of selected emerging parameters on the velocity and temperature fields are plotted and discussed.

Keywords: Upper convected Maxwell fluid, variable viscosity, Thermal stratification, thermal radiation, Melting surface

INTRODUCTION

The first statement of the second law of thermodynamics can be explained in terms of the spontaneous heat flow from a hot to a cold body. This implies that an ice cube must melt on a hot surface rather than become colder. In the industry, radiative heat transfer and thermal radiation are commonly used to describe the science of heat transfer caused by electromagnetic waves. Obvious everyday examples of thermal radiation include the heating effect of sunshine on a clear day. The fact that, when one is standing in front of a fire; the side of the body facing the fire feels much hotter than the back, and so on. Physically, heat flows constantly from our body to the air around us. All bodies constantly emit energy by a process of electromagnetic radiation. The intensity of such energy flux strongly depends on the temperature of the body and the nature of its surface. In order to investigate this phenomenon, Lienhard-IV and Lienhard-V (2008) stated that objects that are cooler than the fire emit much less energy because the energy emission varies as the fourth power of absolute temperature. Sandeep and Sugunamma (2014) considered these facts and reported the effect of thermal radiation on hydromagnetic free convection flow of a viscous incompressible electrically conducting fluid.

The analysis, description, theoretical and experimental studies of boundary layer flow together with heat transfer across a special kind of fluid (i.e. having the properties of elasticity and viscosity when undergoing deformation) as it flows over a horizontal surface has gained the attention of many researchers. The dynamics of such material (*Maxwell fluid*) is a fundamental topic in fluid dynamics and it has attracted the attention of many researchers due to its wide industrial and technical applications. Series of investigations have been carried out towards the understanding of the dynamics of viscoelastic material since the contribution of Maxwell (1867) and Christopher (1993). The Upper Convected Maxwell model can be described as the generalization of the Maxwell material for the case of large deformation using the upper-convected time derivative (also known as Oldyrold derivative) which is the rate of change of some tensor properties of a small parcel of fluid that is written in the coordinate system stretching with the fluid. It is worth pointing out that this concept has not been extended to investigate the dynamics of UCM flow in between melting surface and thermal stratification located at the freestream.

It is a common known fact in rheology that given enough time, even a solid-like material will flow; see Barnes *et al.* (1989). In view of this, it is required to characterize the fluidity of materials under specific flow conditions (i.e. a dimensionless number that incorporates both the elasticity and viscosity of material is required). In the same context, it was reported that at high Deborah number, UCM flow corresponds to solid-like behavior and low Deborah numbers to fluid-like behavior (Steffe, 1996 and Sadeghy et al. 2005). Raju *et al.* (2016) explained the motion of viscoelastic fluid flow in the presence of induced magnetic field when homogeneous and heterogeneous chemical reaction is considered. Recently, Shateyi *et al.* (2015) adopted Chebyshev Spectral Collocation Method to explain entropy generation on magneto hydrodynamics flow and heat transfer of a Maxwell fluid over a stretching sheet.

Internal energy generation can be explained as a scientific method of generating heat energy within a body by chemical, electrical or nuclear process. Natural convection induced by internal heat generation is a common phenomenon in nature. In many situations, there may be appreciable temperature difference between the surface and the ambient fluid. This necessitates the consideration of temperature dependent heat source(s) that may exert a strong influence on the heat transfer characteristics Salem and El-Aziz (2007). Salem and El-Aziz (2008) further stated that exact modeling of internal heat generation or absorption is quite difficult and argued that some simple mathematical models can express its average behavior for most physical situations. Recently, Animasaun *et al.* (2015) reported that when the plastic dynamic viscosity and thermal conductivity of non-Newtonian Casson fluid are considered as temperature dependent, exponentially decaying internal heat generation parameter is an important dimensionless number that can be used to increase velocity and temperature of the fluid as it flows. Motsa and Animasaun (2016) deliberated on the velocity of fluid flow in the presence of space and temperature-dependent internal heat source due to impulsive. Koriko *et al.* (2016) adopted homotopy analysis method to investigate free Convective micropolar fluid flow in the presence of space dependent internal heat source. Effect of this internally generated heat energy on the surface may lead to melting of solid surface.

From the knowledge of kinetic theory of matter, some solids may melt if expose to a high temperature. In an earlier study, the effect of melting on heat transfer was studied by Yin-Chao and Tien (1963) for the Leveque problem. The corresponding effect of melting on forced convection heat transfer was reported in Tien and Yen (1965). In addition, effect of melting on heat transfer between melting body and surrounding fluid qualitatively from the point of view of boundary layer theory was investigated. This contribution to the existing body of knowledge attracted Epstein (1975) to present a note on a systematic method of calculating steady state melting rates in all circumstances involving the melting of solid bodies immersed in streams of warmer fluid of the same material. Many researchers have investigated the effect of melting parameters; and it was revealed that fluid viscosity and thermal conductivity have been assumed to be constant functions of temperature within the boundary layer. However, it is known that physical properties of the fluid may change significantly when expose to internal generated temperature. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affect the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. In a case of melting as reported by Hayat *et al.* (2010a), Fakusako and Yamada (1999), Ishak *et al.* (2010) and Pop *et al.* (2010), it is worth mentioning that temperature of fluid layers at free stream may also have significant effect on the intermolecular forces of Upper Convected Maxwell fluid.

According to Batchelor (1987), Animasaun (2015a) and Meyers *et al.* (2006), it is a well-known fact that properties which are most sensitive to temperature rise are viscosity and thermal conductivity. Recently, Mukhopadhyay (2013) considered this same fact in order to explain stagnation point flow behavior on non-melting surface while Animasaun (2015a) adopted the model and reported the dynamics of unsteady magnetohydrodynamic convective fluid flow with radiation and thermophoresis of particles past a vertical porous plate moving through a binary mixture in an optically thin environment. Motivated by all the works mentioned above, it is of interest to contribute to the body of knowledge by studying the dynamics of UCM fluid flow considering a case in which the influence of temperature on viscosity and thermal conductivity are properly accounted for. In this study, we aim at investigating the dynamics of UCM fluid flow in between melting surface and thermal stratification located at the freestream. This is achieved by modifying and incorporating all the necessary term(s) into the boundary layer equation in-line with boundary layer theory and heat transfer theory. In addition, the objective is to unravel effects of corresponding parameters on the velocity, temperature, shear stress and temperature gradient of UCM fluid flow with variable thermo physical properties. The knowledge will be useful to Engineers and scientist when interested to increase the efficient of UCM flow during industrial processes.

MATHEMATICAL FORMULATION

We consider steady and incompressible Upper Convected Maxwell (UCM) fluid flow with variable thermophysical properties along a melting surface. The flow under consideration is assumed to occupy the domain $0 \le y < \infty$ as shown in Figure 1. Following Dunn and Rajagopal (1995) and Sadeghy *et al.* (2005), an incompressible fluid flow obeying Upper Convected Maxwell model with temperature dependent dynamics viscosity and thermal conductivity; the continuity equation, $x-$momentum equation and the energy equation can be simplified using the usual boundary layer theory approximations and written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)+\frac{Q_o(T_\infty - T_o)}{\rho c_p}e^{-ny\sqrt{\frac{a}{\vartheta}}}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}. \quad (3)$$

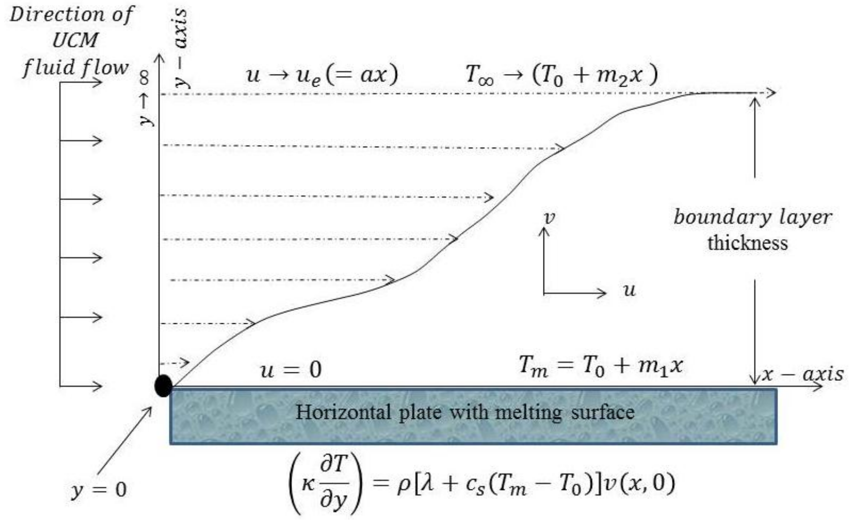

Figure 1: Physical Configuration

The Rosseland approximation in the Energy Equation (3) is a simplification of the radiative transport Equation (RTE) for the case of optically thick media (UCM) as stated in Brewster (1972). The Rosseland approximation for radiation is defined as

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}. \quad (4)$$

Since radiation travels only a short distance before being scattered or absorbed; Equation (4) introduces a new diffusion term into the energy transport equation with a strong temperature dependent diffusion coefficient. Assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor Series and expanding T^4 about T_m; neglecting higher orders.

$$T^4\approx T_m^4+(T-T_m)\left.\frac{dT^4}{dT}\right|_{Tm}+\frac{(T-T_m)^2}{2!}\left.\frac{d^2T^4}{dT^2}\right|_{Tm}+\ldots \quad (5)$$

Neglecting higher order

$$T^4\approx 4T_m^3T-3T_m^4 \quad (6)$$

Also differentiating both sides of Equation (6) partially with respect to T

$$\frac{\partial T^4}{\partial T}\approx 4T_m^3 \quad (7)$$

Hence

$$\frac{\partial q_r}{\partial y}=\frac{\partial}{\partial y}\left(-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}\right)=\frac{\partial}{\partial y}\left(-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial T}\frac{\partial T}{\partial y}\right). \quad (8)$$

Substituting Equation (7) into Equation (8), we obtain

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma T_m^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \quad (9)$$

Substituting Equation (9) into Energy Equation (3)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right)+\frac{Q_o(T_\infty - T_o)}{\rho c_p}e^{-ny\sqrt{\frac{a}{\vartheta}}}+\frac{1}{\rho c_p}\frac{16\sigma T_m^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \quad (10)$$

In this present study, it is important to state that exponential heat source is adopted to account for internal distribution of temperature in the energy equation. This concept can be traced to the idea of Salem and El-Aziz (2007, 2008), Abegunrin *et al.*

(2015a) and Animasaun (2015b). Equation (1), Equation (2) and Equation (10) are subject to the following boundary conditions

$$u=0, \quad \kappa\left(\frac{\partial T}{\partial y}\right)=\rho[\lambda^{*}+c_s(T_m-T_o^{*})]v(x,0), \quad T=T_m \quad \text{at} \quad y=0 \qquad (11a,b,c)$$

$$u \to u_e(=ax), \quad T \to T_\infty \qquad \text{as} \qquad y \to \infty \qquad (12a,b)$$

The symbol κ is the thermal conductivity, λ^{*} is the latent heat of the fluid and c_s is the heat capacity of the solid surface. In order to solve the problem completely in unbounded domains, it is possible to augment the boundary conditions by assuming certain asymptotic structures for the solutions at infinity. The formulation of Equation (11b) states that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required raising solid temperature T_o^{*} to its melting temperature T_m Epstein and Cho (1976). The increase of temperature may also lead to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so the heat transfer rate at the wall may also be affected greatly. Due to this, it is very important to account for the influence of temperature on the thermo-physical properties of UCM fluid as it flows over a melting surface within the boundary layer. However, it is known that physical properties of the fluid may change significantly in the presence of space dependent internal generated temperature. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affect the viscosity of the fluid and so the fluid viscosity can no longer be assumed constant. In order to account for the variation in thermo-physical properties of the fluid as it flows past a horizontal melting surface, it is valid to consider the mathematical model of temperature-dependent viscosity model used in Koriko *et al.* (2015) and Mukhopadhyay (2013) which was developed using the experimental data of Batchelor (1987) together with the mathematical model of temperature-dependent thermal conductivity model used in Animasaun (2015a) as

$$\mu(T)=\mu^{*}[a+b_1(T_w-T)], \;\; \kappa(T)=\kappa^{*}[a+b_2(T-T_\infty)] \text{ valid accurately when } T_w>T_\infty \qquad (13a,b)$$

Since $T_m<T_\infty$, it is necessary to modify Equation (13a,b) as suggested by Koriko *et al.* (2015). and Animasaun (2015c). In this theoretical study, the best way to avoid either increase in the volume or quality of UCM fluid as it flows is to assume parameter $a=1$. These mathematical models together with classical similarity variables for temperature are modified to

$$\mu(T)=\mu^{*}[1+b_1(T_\infty-T)], \qquad \theta(\eta)=\frac{T-T_m}{T_\infty-T_o}, \qquad \kappa(T)=\kappa^{*}[1+b_2(T-T_m)]. \qquad (14a,b,c)$$

Due to the relationship between dynamic viscosity, exponential internal heat source within the fluid domain and free stream temperature of UCM fluid as stated in Equation (14a). Using Equation (15b) and Equation (14b), we obtain

$$T_\infty-T=(1-\theta)\left[T_\infty-T_o\right]-m_1x$$

The idea of Vimala and Loganthan (2015) and Animasaun (2015b) was followed to define thermal stratification (T_m) at the melting wall ($y=0$) and at the free stream ($y\to\infty$) as

$$T_m=T_o+m_1x, \qquad T_\infty=T_o+m_2x. \qquad (15a,b)$$

From these models, it is valid to write the relation of the form

$$b_1(T_m - T_o) = b_1 m_1 x, \quad b_1(T_\infty - T_o) = b_1 m_2 x. \text{(16a,b)}$$

T_o is known as reference temperature. It is worth noticing from Equation (16a,b) that there exist two differences in temperature due to stratification which occurs for all x at fixed point $y = 0$ and the other that occurs for all x as $y \to \infty$. Following Omowaye and Animasaun (2016), it is valid to define temperature dependent viscous parameter ξ as Equation (17a). This assumption is based on the fact that (i) the stress is a linear function of the strain rates as the fluid flows along a surface, and (ii) the UCM fluid is isotropic. The ratio of the two terms in Equations (16a,b) can thus produce the dimensionless thermal stratification parameter (S_t).

$$\xi = b_1(T_\infty - T_o), \qquad b_1(T_m - T_o) = \xi S_t \qquad S_t = {m_1}/{m_2} \qquad \text{(17a,b,c)}$$

Upon using Equation (14) - Equation (17), we obtain

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \vartheta^*[a + \xi - \theta\xi - \xi S_t] - \vartheta^*\xi\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y} - \frac{\sigma B^2}{\rho}u, \quad (18)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa^*[a + \theta\varepsilon](T_\infty - T_o)}{\rho c_p}\frac{\partial^2 \theta}{\partial y^2} + \frac{\kappa^* b_2}{\rho c_p}(T_\infty - T_o)^2\left(\frac{\partial \theta}{\partial y}\right)^2$$

$$+\frac{Q_o(T_\infty - T_o)}{\rho c_p}e^{-ny\sqrt{a/\vartheta}} + \frac{1}{\rho c_p}\frac{16\sigma T_m^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \quad (19)$$

The dimensionless governing equation is obtained by using the following non-dimensional quantities

$$v = -\frac{\partial \psi}{\partial x}, \qquad u = \frac{\partial \psi}{\partial y}, \qquad \eta = y\left(\frac{a}{\vartheta}\right)^{\frac{1}{2}}, \qquad \psi = xf(\eta)\sqrt{a\vartheta} \qquad \text{(20a,b)}$$

It is important to note that Equations (20a,b) automatically satisfy continuity Equations (1). Then, Eqs. (18), (19), (11) and (12) becomes

$$[1 + \xi - \theta\xi - \xi S_t - \beta f^2]\frac{d^3 f}{d\eta^3} + \left(f - \xi\frac{d\theta}{d\eta} + 2\beta f\frac{df}{d\eta}\right)\frac{d^2 f}{d\eta^2} - \left(\frac{df}{d\eta} + M\right)\frac{df}{d\eta} = 0, \quad (21)$$

$$\left[1 + \varepsilon\theta + \frac{4}{3N}\right]\frac{d^2\theta}{d\eta^2} + \varepsilon\frac{d\theta}{d\eta}\frac{d\theta}{d\eta} - P_r S_t\frac{df}{d\eta} - P_r\theta\frac{df}{d\eta} + P_r f\frac{d\theta}{d\eta} + P_r\gamma e^{-n\eta} = 0. \quad (22)$$

The corresponding dimensionless boundary conditions take the form

$$\frac{df}{d\eta} = 0, \qquad m\frac{d\theta}{d\eta} + P_r f = 0, \qquad \theta = 0 \qquad \text{at} \qquad \eta = 0 \qquad (22)$$

$$\frac{df}{d\eta} \to 1, \qquad \theta \to (1 - S_t) \qquad \text{as} \qquad \eta \to \infty. \qquad (23)$$

Dimensionless viscoelastic parameter (Deborah number) $\beta = \lambda a$, temperature dependent thermal conductivity parameter $\varepsilon = b_2(T_\infty - T_o)$, magnetic field parameter $M = \sigma B^2 / a\rho$, coefficient of thermal diffusivity $\alpha = \kappa / \rho c_p$, Prandtl number $P_r = c_p \mu / \kappa$, thermal radiation parameter $N = \kappa\kappa^* / (4\sigma T_m^3)$, heat source parameter $\gamma = Q_o / \rho c_p a$ and melting parameter $m = (T_\infty - T_o) c_p / [\lambda^* + c_s (T_m - T_o^*)]$.

METHOD OF SOLUTION

Numerical solutions of the ordinary differential Equation (21) and Equation (22) subject to Neumann boundary conditions (22) and (23) are obtained using classical Runge-Kutta method with shooting techniques. The BVP cannot be solved on an infinite interval and it would be impractical to solve it for even a very large finite interval. In this study, we imposed the infinite boundary condition at a finite point $\eta_\infty = 4$. The set of coupled nonlinear ordinary differential equations along with boundary conditions have been reduced to a system of five simultaneous equations of first order for five unknowns following the method of superposition Na (1979). In order to integrate the corresponding IVP the values of $f(0)$, $f''(0)$, and $\theta'(0)$ are required, but no such values exist after the non-dimensionalization of the boundary conditions (Equations 11 and 12). The suitable guess values for $G_1 = f''(0)$, $G_2 = \theta'(0)$ and $G_3 = f(0)$ are chosen and then integration is carried out. In this method of solution, Newton-Raphson method is embedded as root finding of the non-linear equations of the corresponding systems of five first-order ordinary differential equations. The five initial conditions at ($\eta = 0$) can now be defined as column vector with five elements as

$$Guess = \left[(\Pr G_3 + mG_2); 0; G_1; 0; G_2 \right].$$

The calculated values for $f(\eta)$ and $\theta(\eta)$ at $\eta = 4$ are compared with the known boundary conditions in Equation (23) and the estimated values $f''(0)$ and $\theta'(0)$ are adjusted to give a better approximation of the solution. Series of values for $f''(0)$ and $\theta'(0)$ are considered and applied with fourth-order classical Runge-Kutta method using step size $\Delta\eta = h = 0.01$. The above procedure is repeated until asymptotically converged results are obtained within a tolerance level of 10^{-5}.

DISCUSSION

The numerical computations have been carried out for various values of temperature dependent viscous parameter, stratification parameter, Deborah number, magnetic field parameter, thermal radiation parameter, temperature dependent thermal conductivity parameter, Prandtl number, space dependent heat source parameter, intensity of heat distribution on space parameter and melting parameter using numerical scheme discussed in the previous section. In order to illustrate the results graphically, the numerical values are plotted in Figures 2 - 7. The variations of $f(\eta)$ and $f'(\eta)$ along η with different values of ξ are plotted in Figure 2 and 3 respectively. It is seen that an increase in the magnitude of ξ leads to the enhancement of transverse velocity and velocity profiles. It is further noticed that, increase in the magnitude of ξ has no significant effect on velocity profiles near the melting wall. As shown in Figure 2, the parameter ξ has an evident effect on $f(\eta)$ that the larger the value of ξ is, the greater the velocity. It is also observed that temperature dependent viscous parameter has profound effect on transverse velocity profiles within the fluid domain compare to that of horizontal velocity profiles. This can be traced to the high value assigned to melting parameter. From Equation (22), it easy to deduce that melting parameter is a multiple of temperature gradient term. It indicates that, as magnitude of ξ increases at a constant value of b_1, there is a corresponding increase in temperature difference of $(T_m - T_o)$.

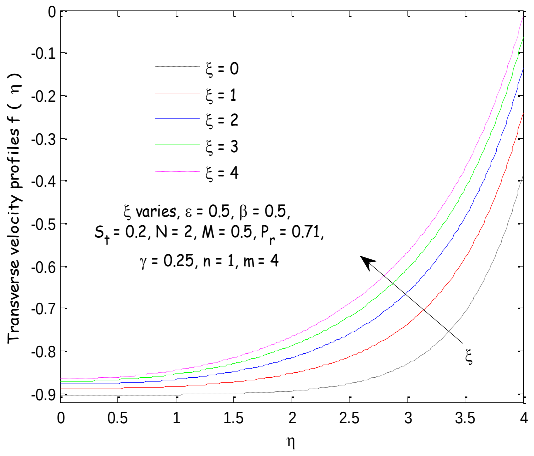

Figure 2: Increasing effect of temperature dependent variable fluid viscosity parameter over transverse velocity profiles of Upper Convected Maxwell fluid

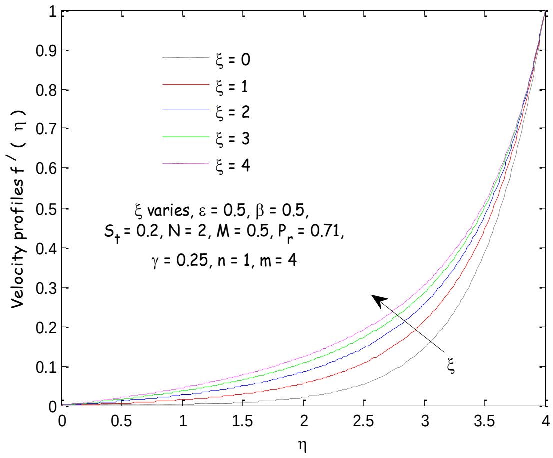

Figure 3: Increasing effect of temperature dependent variable fluid viscosity parameter over transverse velocity profiles of Upper Convected Maxwell fluid

profiles near the free stream. Physically, the temperature of UCM fluid within few layers near freestream is almost the same; this account for the negligible increasing effect of ξ over velocity profiles. In such a situation, the flow velocity approaches to the possible maximum value.

In the presence of thermal radiation and high magnitude of melting at the wall, the effect of temperature dependent variable fluid viscosity (ξ) over shear stress profiles is further unravel the dynamics of UCM fluid flow over melting surface, the effect of melting parameter (m) and temperature dependent variable fluid viscosity parameter (ξ) over local skin friction coefficient $f^{//}(0)$ which is proportional to local heat transfer rate is shown in Figure 5.

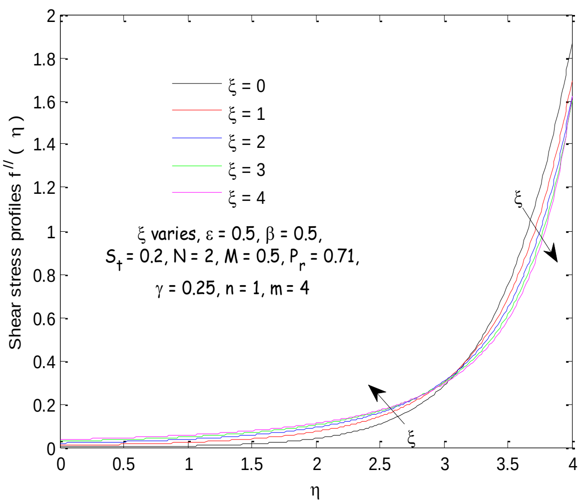

Figure 4: Increasing effect of temperature dependent variable fluid viscosity parameter over Shear stress profiles of Upper Convected Maxwell fluid

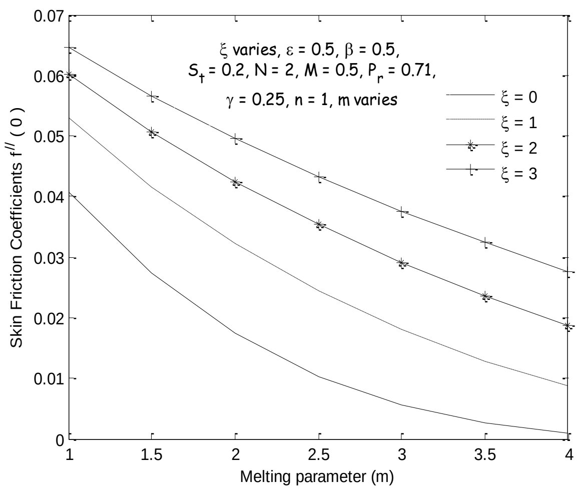

Figure 5: Increasing effect of temperature dependent variable fluid viscosity parameter and melting parameter over Shear stress profiles of Upper Convected Maxwell fluid at the wall.

It is observed that the rate of increase in $f^{//}(0)$ with (ξ) is significant at higher value of melting parameter. The effect of both parameters on shear stress profiles near freestream is presented in Table 1.

Table 1: Increasing effect of temperature dependent variable fluid viscosity parameter (ξ) and melting parameter (m) over Shear stress profiles $f^{//}(\eta=4)$ of Upper Convected Maxwell fluid near the free stream.

	$f^{//}(\eta=4)$ **when** $(\xi=0)$	$f^{//}(\eta=4)$ **when** $(\xi=1)$	$f^{//}(\eta=4)$ **when** $(\xi=2)$	$f^{//}(\eta=4)$ **when** $(\xi=3)$
$m=1$	0.800497489277775	0.852705981783623	0.915866357976491	0.983437834478091
$m=1.5$	0.917299421257627	0.953179275701427	1.005858881864090	1.065568882695260
$m=2$	1.047440672263030	1.061787074416630	1.100999956353010	1.151835743690170
$m=2.5$	1.198445372440900	1.184445440836170	1.205548863444140	1.246373506726640
$m=3$	1.377604438847530	1.324973870536900	1.326217958134970	1.353667166929880
$m=3.5$	1.593361976072640	1.491424281205490	1.460604532294710	1.471820423608440
$m=4$	1.866834452426350	1.694291592072390	1.622345361664350	1.603310965833320

It is easy to deduce from Table 1 that at each value of (ξ), the coefficient $f^{//}(\eta=4)$ is an increasing function of melting parameter (m). Physically, the significant increase in the corresponding value of $f^{//}(\eta=4)$ due to increasing in the magnitude of melting parameter at a constant value of (ξ) can be traced to the influence of substantial heat energy which exists at the free stream. The influence of thermal radiation is illustrated in Figures 6 and 7. It is noticed that an increase in the radiation parameter results in decrease of the temperature profiles of UCM within the boundary layer. It is also noticed that the temperature gradient also decreases significantly within the fluid domain with the increasing value of the radiation parameter N. The effect of radiation parameter (N) is to reduce the temperature significantly in the flow region. Thermal radiation is found to be energy transfer by the emission of electromagnetic waves which carry energy away as the fluid flows. Since the increase in radiation parameter means increase in the rate at which heat energy is released from the flow region; this account for decrease in temperature distribution as the magnitude of parameter N increases.

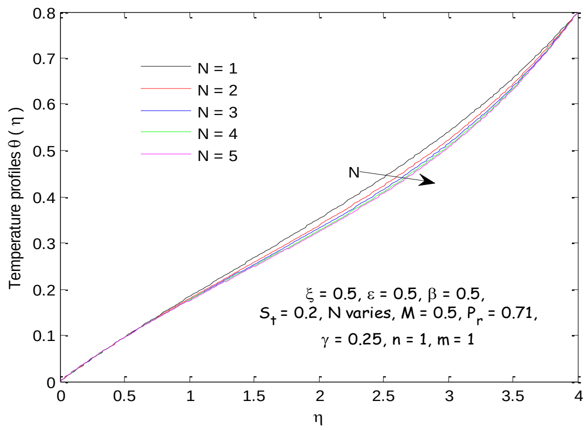

Figure 6: Increasing effect of thermal radiation parameter over temperature profile of Upper Convected Maxwell fluid

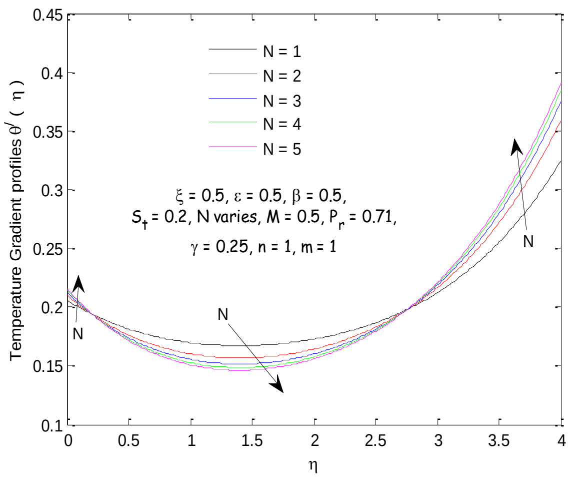

Figure 7: Increasing effect of thermal radiation parameter over temperature gradient profiles of Upper Convected Maxwell fluid

CONCLUSION

The similarity solutions of steady UCM fluid flow over melting surface have been studied. The corresponding effects of thermal radiation and variation in viscosity due to temperature have properly investigated. Results for the skin friction coefficient, local Nusselt number, transverse velocity profiles, velocity profiles as well as temperature profiles are presented for different values of the governing parameters. Effects of the melting parameter, temperature dependent viscous parameter, temperature dependent thermal conductivity parameter, Deborah number and Prandtl number on the flow and heat transfer characteristics are thoroughly examined. The main points of the present study can be summed up as follows:

1. the modified version of temperature dependent variable viscosity and thermal conductivity of UCM gives better and correct analysis of dynamics of fluid flow along horizontal melting heat transfer surface. This conclusion is based on the fact that boundary conditions of temperature as in the case of melting heat transfer satisfy $T_m < T_\infty$.

2. local skin friction coefficient $f^{//}(0)$ increases with an increase in the magnitude of temperature dependent variable fluid viscosity.

3. in the industry, transverse velocity and longitudinal velocity of Upper Convected Maxwell fluid flow over melting surface (subject to thermal stratification) can be increased with an increase in the magnitude of temperature dependent variable fluid viscosity.

4. the thermal radiation parameter decreases the temperature within UCM significantly within the fluid domain in the thin boundary layer. An increase in thermal radiation parameter causes significant increase in temperature gradient near the free stream $\eta = 4$.

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