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COMPARISON OF NON-PARAMETRIC METHODS FOR PAIRED SAMPLE DATA

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ABSTRACT
This study was on the comparison of non-parametric methods for paired sample data. A sample of fifty healthy adults' body temperatures were used to analyze the statistical methods used. The study was aimed at comparing a rank based test statistic for testing the equality of two population medians and comparing the Oyeka and Umeh method with sign test and Wilcoxon signed rank test. The three statistical methods were compared with each other to know which was better by testing the efficiency, standard error and minimum variance. The standard error of the proposed method Oyeka and Umeh method was 0.947 as compare to that of Wilcoxon signed rank which was 13.207. The efficiency of the proposed test statistic relative to the Wilcoxon sign ranked test is 175.12, which was greater than 1, therefore the proposed method is more efficient than the Wilcoxon signed rank and sign tests. It also has an added advantage over the two tests because it uses all the available information in the data.
Keywords: Non-parametric, Rank, Sign test, Wilcoxon signed rank test, Paired Sample Data

INTRODUCTION
The non-parametric procedure is used when the usual assumptions of continuity and normality of the parametric method are not satisfied.(Andy 2009) Non-parametric tests are also called distribution free tests. It can be used when testing nominal or ordinal variables and the assumptions of parametric test have not been met. There is a wide range of methods that can be used in different circumstances, some of the non-parametric method that are readily available are the sign test and the Wilcoxon signed rank test (Lindgren 1968; Siegel and Castellan 1988).Non-parametric models differ from parametric models in that the model structure is not specified a priori but is instead determined from data. The term non-parametric does not imply that such models completely lack parameters but that the number and nature of the parameters are flexible and not fixed in advance (Siegel and Castellan 1988; Gibbons 1971).Pairing seeks to reduce variability in order to make more precise comparisons with fewer subjects. Two measurements are paired when they come from the same observational unit. The best way to determine whether data are paired is to identify the natural link between the two measurements. However, instead of taking the differences between pairs of observations and using these differences with the sign test or taking the absolute values of these differences assigning them ranks and applying the Wilcoxon signed rank test, ranks are first assigned to each element in the pair of observation and the difference between the ranks are taken (Onyeka and Umeh 2012). Also, if the difference is greater, equal or less than zero, values (1, 0, -1) are assigned respectively. The three statistical tests are then compared to know which is better. A random sample of 50 healthy adults was used to test the three statistical methods used, their temperatures were taken at a particular time for

two days. The observation of day 1 was paired with the observation of day 2 for each subject.

The objectives of this research are to test the performances and efficiency of the three statistical methods used for pair sample data; i.e to test whether the performances of Onyeka and Umeh method is better and efficient than Sign test and Wilcoxon signed rank test.

METHODOLOGY

The data used was researched by Dr. Steven Wasserman, Dr. Philip Mackowiak, and Dr. Myron Levine from University of Maryland, U.S.A. as will be shown in the study (Mario 2011)

SIGN TEST: The sign test is a relatively simple non-parametric procedure for testing hypothesis about the central tendency of a non-normal probability distribution. It is a non-parametric version of the paired t test used when pairing is appropriate and when normality assumption required for the t- test. It allocates a sign either positive (+) or negative (-), to each observation according to whether it is greater or less than some hypothesized value. It is also based on differences d, between the pairs of observation. The information used by the sign test from this difference is the sign of d (+ or -), i.e. paired sample sign test only uses the sign of paired difference ranks.(Gibbons 1971; Mario 2011; Siegel and Castellan 1988)

WILCOXON SIGNED RANK TEST: Wilcoxon signed rank test was designed by Frank Wilcoxon (Wilcoxon 1945). It account for the magnitude of the observations. Paired sample Wilcoxon signed rank test uses magnitude and sign of the paired difference ranks. It (also referred to as the Wilcoxon matched pairs signed ranks test) is designed for use with repeated measures.
(Gibbons 1971; Mario 2011; Siegel and Castellan 1988)

OYEKA AND UMEH METHOD: Oyeka, I.C.A. and Umeh, E.U. (Oyeka and Umeh, 2012) developed a method in which ranks are first assigned either from the smallest to the largest or from the largest to the smallest member of the paired observations rather than taking the difference between pairs of observation and applying the sign test or assigning ranks to the absolute values of the difference and applying the Wilcoxon signed rank test. A rank based test statistic is developed using the ranks assigned first to the pairs of observation, which is used to test the equality of two population medians. Rank r_{i1} is assigned to the first pair and r_{i2} is assigned to the second pair as follows;

$$r_{i1} = \begin{cases} 1 \; if \; x_{i2} > x_{i1} \\ 1.5 \; if \; x_{i2} = x_{i1} \\ 2 \; if \; x_{i2} < x_{i1} \end{cases} \quad \text{for I = 1, 2, . . . n}$$

$$r_{i2} = \begin{cases} 1 \; if \; x_{i1} > x_{i2} \\ 1.5 \; if \; x_{i1} = x_{i2} \\ 2 \; if \; x_{i1} < x_{i2} \end{cases} \quad \text{for I = 1, 2, . . . n}$$

Let $r_i = r_{i2} - r_{i1}$ ….. (1)

Define $U = \begin{cases} 1 \;\; if \;\; r_i > 0 \\ 0 \;\; if \;\; r_i = 0 \\ -1 \;\; if \;\; r_i < 0 \end{cases}$ ….. (2)

For I = 1, 2 …n

If paired observations are randomly drawn from the population, then $\pi^+, \pi^0, \; and \; \pi^-$ are respectively the probabilities that the second elements(x_{i2}) in the pairs of the observations are on the average greater than equal to or less than the first elements x_{i1} in the pairs of the observations. (Oyeka; Ebuh ; Nwankwo ; Obiora-Ilouno; Ibeakuzie and Utazi , 2010).

Let $\pi^+ = P(U_i = 1); \pi^0 = P(U_i = 0)$;

$\pi^- = P(U_i = -1)$

Where $\pi^+ + \pi^0 + \pi^- = 1$ ………… (3)

Let $W = \sum_{i=1}^{n} U_i$ ………………………....(4)

$$E(U_i) = \sum U \, P(U_i)$$

$$E(U_i) = 1(\pi^+) + 0(\pi^0) + (-1)(\pi^-)$$

$E(U_i) = \pi^+ - \pi^-$ ……...…...…....(5)

$Var(U_i) = E(U_i)^2 - \left(E(U_i)\right)^2$ ……… (6)

$Var(U_i) = \pi^+ + \pi^- - (\pi^+ - \pi^-)^2$

The estimated probabilities are;

$\hat{\pi}^+ = \frac{f^+}{n}; \hat{\pi}^0 = \frac{f^0}{n}; \hat{\pi}^- = \frac{f^-}{n}$.(7)

Where $f^{+}, f^{0}\ and\ f^{-}$ are the respective the numbers of $1^{s}, 0^{s}$and -1^{s} in the frequency distribution of size n of the numbers in U_i ,

i = 1, 2…n

Let: E (W) = $\sum_{i=1}^{n} E(U_i)$ = n$(\pi^{+} - \pi^{-})$ (8)

Var (W) = $\sum_{i=1}^{n} var(u_i)$ =

n$(\pi^{+} + \pi^{-} - (\pi^{+} - \pi^{-})^2)$ …………..... (9)

Equation (8) is estimated as:

$$\hat{\pi}^{+} - \hat{\pi}^{-} = \frac{w}{n} \quad \text{………………} (10)$$

From equation (10) in equation (9), the sample estimate of the variance of W becomes

$$\text{Var (W)} = n(\hat{\pi}^{+} + \hat{\pi}^{-}) - \frac{w^2}{n} \quad (11)$$

If the population medians are equal, then $\pi^{+} - \pi^{-}$ would be expected to be equal to zero. Assuming the population medians differ by some constant values, θ. i.e.

$\pi^{+} - \pi^{-} = \theta$. therefore, a null hypothesis for the research;

$$H_0: \pi^{+} - \pi^{-} = \theta$$

$$H_1: \pi^{+} - \pi^{-} \neq \theta \quad (12)$$

If H_0 is true then the test statistic; $X^2 = \frac{(W-n\theta)^2}{Var(W)} = \frac{(W-n\theta)^2}{n(\hat{\pi}^{+} + \hat{\pi}^{-}) - \frac{w^2}{n}}$ has approximately the chi-square distribution with one (1) degree of freedom for sufficiently large n and may be used to test the null hypothesis of equation (12). The null hypothesis is rejected at the level ofSignificance if $X^2 \geq X^2{}_{1-\alpha/2,\ 1}$

Also, if the constant value $\theta = 0$, then;

$$X^2 = \frac{W^2}{Var(W)} = \frac{W^2}{n(\hat{\pi}^{+} + \hat{\pi}^{-})} \quad (13)$$

H_0 Is rejected, if $X^2 \geq X^2{}_{1-\alpha/2,\ 1}$

MINIMUM VARIANCE

Minimum Variance is an unbiased estimator that has lower variance than any unbiased estimator for all possible values of the parameter. Variance is the square of standard deviation.

STANDARD ERROR

The standard error is the standard deviation of the sampling distribution of a statistic. Standard error is a statistical term that measures the accuracy with which a sample represents a population. In statistics, a sample mean deviates from the actual mean of a population; this deviation is the standard error. The standard error is denoted by S_E. $S_E = \frac{S.D}{\sqrt{n}}$; where S.D is the standard deviation.

EFFICIENCY

The efficiency of a test tells us the power of such test. It is a term used in the comparison of various statistical procedures. The relative efficiency of two procedures is the ratio of their efficiencies. It is often defined using the variance, the statistical method with the minimum variance is said to be more efficient. The relative efficiency of two sets is a measure of the relative power of two sets. To prove that the proposed test statistic “W” is generally more powerful than the Wilcoxon signed rank sum test statistic that ignore zero absolute differences. To show that “W” is relatively more efficient that “T” for a specified sample size. Note that the variance of “T” for a given sample size “n” is

$$\text{Var (T)} = \frac{n(n+1)(2n+1)}{24}$$

$$\text{Var (W)} = n(\hat{\pi}^{+} + \hat{\pi}^{-}) - \frac{w^2}{n}$$

The efficiency of “W” relative to “T” is calculated as;

$$\text{RE(W,T)} = \frac{Var(T)}{Var(W)} = \frac{n(n+1)(2n+1)}{24n(\pi^{+} + \pi^{-} - (\pi^{+} - \pi^{-})^2)} \geq \frac{(n+1)(2n+1)}{24(1-\pi^{0})}$$

Note that $\pi^{+} + \pi^{-} + \pi^{0} = 1$

Hence, $\pi^{+} + \pi^{-} = 1 - \pi^{0}$;

$(\pi^{+} - \pi^{-})^2 \geq 0$

RE$(W,T) > 1$; For all n ≥ 1

Therefore, the proposed test statistic is relatively more efficient than the Wilcoxon signed rank test statistic.

DATA ANALYSIS

SIGN TEST

HYPOTHESIS

H_0: The distribution of the two groups is the same

H_1:The distribution is different

Level of significance $\alpha = 0.05$

TEST STATISTICS

The sample size is greater than 25 (i.e. Large Samples), the test statistics Z is;

$$Z = \frac{(x+0.5)-(0.5n)}{\frac{\sqrt{n}}{2}}$$

Decision Rule: Reject H_0 if Z calculated > Z tabulated, otherwise do not reject.

TABLE 1: SIGN TEST

SUBJECT	x_{i1}	x_{i2}	SIGN of $(x_{i2} - x_{i1})$
1	98.0	98.6	+
2	98.8	98.6	−
3	98.0	98.0	0
4	98.8	98.0	−
5	98.8	99.0	+
6	97.6	98.4	+
7	98.6	98.4	−
8	98.6	98.4	−
9	98.8	98.4	−
10	98.0	98.6	+
11	98.2	98.6	+
12	98.0	98.8	+
13	98.0	98.6	+
14	97.0	97.0	0
15	97.2	97.0	−
16	98.2	98.8	+
17	98.1	97.3	−
18	98.2	98.7	+
19	98.5	97.4	−
20	98.5	98.9	+

21	99.0	98.6	−
22	98.0	99.5	+
23	97.0	97.5	+
24	97.3	97.3	0
25	97.3	97.6	+
26	98.1	98.2	+
27	97.8	98.7	+
28	99.0	99.4	+
29	97.6	98.2	+
30	97.4	98.0	+
31	98.0	98.6	+
32	97.4	98.6	+
33	98.0	97.2	−
34	98.6	98.6	0
35	98.6	98.2	−
36	98.4	98.0	−
37	97.0	97.8	+
38	98.4	98.4	0
39	99.0	98.6	−
40	98.0	98.6	+
41	99.4	99.0	−
42	97.8	97.6	−
43	98.2	96.9	−
44	99.2	97.6	−
45	99.0	97.1	−
46	97.7	97.9	+
47	98.2	98.4	+
48	98.2	97.3	−
49	98.8	98.0	−
50	98.1	97.5	−

$n = 45, \ N_+ = 24, \ N_- = 21, x = 21$

$$Z = \frac{(x+0.5)-(0.5n)}{\frac{\sqrt{n}}{2}} \ ; \ Z = \frac{(21+0.5)-0.5(45)}{\frac{\sqrt{45}}{2}};$$

$$Z = -0.2981 \qquad \text{.................................(14)}$$

Conclusion: Since 0.2981 < 1.96, we do not reject the null hypothesis and conclude that the distribution of the two groups are the same.

WILCOXON SIGNED RANK TEST

HYPOTHESIS

H_0: There is no difference between the Temperature at day 1 and day 2

H_1: There is difference

Level of significance $\alpha = 0.05$

TEST STATISTICS

The test statistics Z for sample size greater than 30 (n > 30) is;

$$Z = \frac{W_+ - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}} \qquad \text{................................ (15)}$$

Decision Rule: RejectH_0, if $Z_{cal} > Z_{\alpha/2}$,

Otherwise do not reject.

TABLE 2: WILCOXON SIGNED RANK TEST

SUBJECT	x_{i1}	x_{i2}	$d_i = (x_{i2} - x_{i1})$	$\|d_i\|$	r_i	Signed rank
1	98.0	98.6	+0.6	0.6	26	+26
2	98.8	98.6	−0.2	0.2	5.5	−5.5
3	98.0	98.0	0	0	-	-
4	98.8	98.0	−0.8	0.8	34	−34
5	98.8	99.0	+0.2	0.2	5.5	+5.5
6	97.6	98.4	+0.8	0.8	34	+34
7	98.6	98.4	−0.2	0.2	5.5	−5.5
8	98.6	98.4	−0.2	0.2	5.5	−5.5
9	98.8	98.4	−0.4	0.4	15	−15
10	98.0	98.6	+0.6	0.6	26	+26
11	98.2	98.6	+0.4	0.4	15	+15
12	98.0	98.8	+0.8	0.8	34	+34
13	98.0	98.6	+0.6	0.6	26	+26
14	97.0	97.0	0	0	-	-
15	97.2	97.0	−0.2	0.2	5.5	−5.5
16	98.2	98.8	+0.6	0.6	26	+26
17	98.1	97.3	−0.8	0.8	34	−34
18	98.2	98.7	+0.5	0.5	20.5	+20.5
19	98.5	97.4	−1.1	1.1	40	−40
20	98.5	98.9	+0.4	0.4	15	+15
21	99.0	98.6	−0.4	0.4	15	−15
22	98.0	99.5	+1.5	1.5	43	+43
23	97.0	97.5	+0.5	0.5	20.5	+20.5
24	97.3	97.3	0	0	-	-
25	97.3	97.6	+0.3	0.3	10	+10
26	98.1	98.2	+0.1	0.1	1	+1
27	97.8	98.7	+0.9	0.9	38.5	+38.5
28	99.0	99.4	+0.4	0.4	15	+15
29	97.6	98.2	+0.6	0.6	26	+26

30	97.4	98.0	+0.6	0.6	26	+26
31	98.0	98.6	+0.6	0.6	26	+26
32	97.4	98.6	+1.2	1.2	41	+41
33	98.0	97.2	−0.8	0.8	34	−34
34	98.6	98.6	0	0	-	-
35	98.6	98.2	−0.4	0.4	15	−15
36	98.4	98.0	−0.4	0.4	15	−15
37	97.0	97.8	+0.8	0.8	34	+34
38	98.4	98.4	0	0	-	-
39	99.0	98.6	−0.4	0.4	15	−15
40	98.0	98.6	+0.6	0.6	26	+26
41	99.4	99.0	−0.4	0.4	15	−15
42	97.8	97.6	−0.2	0.2	5.5	−5.5
43	98.2	96.9	−1.3	1.3	42	−42
44	99.2	97.6	−1.6	1.6	44	−44
45	99.0	97.1	−1.9	1.9	45	−45
46	97.7	97.9	+0.2	0.2	5.5	+5.5
47	98.2	98.4	+0.2	0.2	5.5	+5.5
48	98.2	97.3	−0.9	0.9	38.5	−38.5
49	98.8	98.0	−0.8	0.8	34	−34
50	98.1	97.5	−0.6	0.6	26	−26

$W_+ \rightarrow$ Sum of ranks with positive sign $= 26 + 5.5 + 34 + \ldots + 5.5 + 5.5 = 546$

n = 45;

$Var(T) = \frac{n(n+1)(2n+1)}{24}$; $Var(T) = \frac{45(45+1)(2(45)+1)}{24}$; Var(T) = 7848.75

$Z = \frac{W_+ - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$; $Z = \frac{546 - \frac{45(45+1)}{4}}{\sqrt{\frac{45(45+1)(2(45)+1)}{24}}}$; Z = 0.3217

Conclusion: Since $Z_{cal} < Z_{\alpha/2}(1.96)$, we do not reject H_0 and conclude that there is no significant difference.

TABLE 3: OYEKA AND UMEH METHOD

SUBJECT	x_{i1}	x_{i2}	r_{i1}	r_{i2}	$r_i = r$	$U_i = \begin{cases} 1\ if\ r \\ 0\ if\ r \\ -1\ if \end{cases}$	$d_i = (x_{i2}$	$\lvert d_i \rvert$	Rankof $\lvert d_i \rvert$
1	98.0	98.6	1	2	1	1	+0.6	0.6	26
2	98.8	98.6	2	1	−1	−1	−0.2	0.2	5.5
3	98.0	98.0	1.5	1.5	0	0	0	0	-
4	98.8	98.0	2	1	−1	−1	−0.8	0.8	34
5	98.8	99.0	1	2	1	1	+0.2	0.2	5.5

6	97.6	98.4	1	2	1	1	+0.8	0.8	34
7	98.6	98.4	2	1	−1	−1	−0.2	0.2	5.5
8	98.6	98.4	2	1	−1	−1	−0.2	0.2	5.5
9	98.8	98.4	2	1	−1	−1	−0.4	0.4	15
10	98.0	98.6	1	2	1	1	+0.6	0.6	26
11	98.2	98.6	1	2	1	1	+0.4	0.4	15
12	98.0	98.8	1	2	1	1	+0.8	0.8	34
13	98.0	98.6	1	2	1	1	+0.6	0.6	26
14	97.0	97.0	1.5	1.5	0	0	0	0	-
15	97.2	97.0	2	1	−1	−1	−0.2	0.2	5.5
16	98.2	98.8	1	2	1	1	+0.6	0.6	26
17	98.1	97.3	2	1	−1	−1	−0.8	0.8	34
18	98.2	98.7	1	2	1	1	+0.5	0.5	20.5
19	98.5	97.4	2	1	−1	−1	−1.1	1.1	40
20	98.5	98.9	1	2	1	1	+0.4	0.4	15
21	99.0	98.6	2	1	−1	−1	−0.4	0.4	15
22	98.0	99.5	1	2	1	1	+1.5	1.5	43
23	97.0	97.5	1	2	1	1	+0.5	0.5	20.5
24	97.3	97.3	1.5	1.5	0	0	0	0	-
25	97.3	97.6	1	2	1	1	+0.3	0.3	10
26	98.1	98.2	1	2	1	1	+0.1	0.1	1
27	97.8	98.7	1	2	1	1	+0.9	0.9	38.5
28	99.0	99.4	1	2	1	1	+0.4	0.4	15
29	97.6	98.2	1	2	1	1	+0.6	0.6	26
30	97.4	98.0	1	2	1	1	+0.6	0.6	26
31	98.0	98.6	1	2	1	1	+0.6	0.6	26
32	97.4	98.6	1	2	1	1	+1.2	1.2	41
33	98.0	97.2	2	1	−1	−1	−0.8	0.8	34
34	98.6	98.6	1.5	1.5	0	0	0	0	-
35	98.6	98.2	2	1	−1	−1	−0.4	0.4	15
36	98.4	98.0	2	1	−1	−1	−0.4	0.4	15
37	97.0	97.8	1	2	1	1	+0.8	0.8	34
38	98.4	98.4	1.5	1.5	0	0	0	0	-
39	99.0	98.6	2	1	−1	−1	−0.4	0.4	15
40	98.0	98.6	1	2	1	1	+0.6	0.6	26
41	99.4	99.0	2	1	−1	−1	−0.4	0.4	15
42	97.8	97.6	2	1	−1	−1	−0.2	0.2	5.5
43	98.2	96.9	2	1	−1	−1	−1.3	1.3	42
44	99.2	97.6	2	1	−1	−1	−1.6	1.6	44
45	99.0	97.1	2	1	−1	−1	−1.9	1.9	45
46	97.7	97.9	1	2	1	1	+0.2	0.2	5.5
47	98.2	98.4	1	2	1	1	+0.2	0.2	5.5
48	98.2	97.3	2	1	−1	−1	−0.9	0.9	38.5
49	98.8	98.0	2	1	−1	−1	−0.8	0.8	34
50	98.1	97.5	2	1	−1	−1	−0.6	0.6	26

n=50, $f^+ \rightarrow number\ of\ 1^s = 24, f^- \rightarrow number\ of\ -1^s = 21,\ f^0 \rightarrow number\ of\ 0^s = 5$

$$\hat{\pi}^+ = \frac{f^+}{n} = \frac{24}{50} = 0.48\ ; \hat{\pi}^- = \frac{f^-}{n} = \frac{21}{50} = 0.42\ ; \hat{\pi}^0 = \frac{f^0}{n} = \frac{5}{50} = 0.1$$

W = $f^+ - f^-$; W = 24 – 21 ; W = 3

Var(W) = $n(\hat{\pi}^+ + \hat{\pi}^-) - \frac{w^2}{n}$; Var (W) = 44.82

$$\chi^2 = \frac{W^2}{Var(W)}; \qquad \chi^2 = 0.201 \qquad (16)$$

Conclusion: Since 0.201 < 5.02, we do not reject the null hypothesis and conclude that there is no difference between the population medians.

MINIMUM VARIANCE: The statistical test with the minimum variance is said to be a better estimator. Variance of Wilcoxon Signed Rank Test is 7848.75 which is greater than the variance of the Proposed Method of 44.82. Hence, the proposed method has the minimum variance.

STANDARD ERROR

The standard error of Wilcoxon Signed Rank Test is

$$S_E = \frac{S.D}{\sqrt{n}};\ \text{where} S.D = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

$$S.D = \sqrt{\frac{45(45+1)(2(45)+1)}{24}} = 88.593;$$

$$S_E = \frac{88.593}{\sqrt{45}} = 13.207$$

The standard error of the proposed method is;

$$S.D = \sqrt{n(\hat{\pi}^+ + \hat{\pi}^-) - \frac{w^2}{n}} = \sqrt{50(0.48 + 0.42) - \frac{3^2}{50}} = 6.695$$

$$S_E = \frac{6.695}{\sqrt{50}} = 0.947$$

EFFICIENCY

The efficiency of the proposed test statistic "W" relative to Wilcoxon signed rank test "T"

RE(W,T) = $\frac{Var(T)}{Var(W)}$ RE(W, T) $= \frac{7848.75}{44.82}$

RE(W,T) = 175.12

RESULTS AND DISCUSSION

The test of normality was first carried out to ensure that the data does not violate the non-normal assumption. The Kolmogorov-Smirnov test gave a p-value of 0.036 for day1 and 0.003 for day2, which was less than 0.05 at α level of significance. Therefore, it was concluded that the data did not follow a normal distribution.

The sign test analysis gave a Z-value of 0.2981 which was compared to the tabulated $Z\alpha_{/2}$ value of 1.9.Since 0.2981 < 1.9; it implied that the distribution of the two groups were the same. The SPSS analysis has a Z-value of -0.298. The p-value was 0.766 which was greater than the alpha level 0.05; hence, the distribution are the same.

The Wilcoxon signed rank test had a Z-value of 0.3217. Since 0.3217 < 1.9, it implied there was no significant difference between the temperature at day one and day two. The Z-value from the SPSS output is -0.323. The P-value of 0.747 was greater than the alpha level of 0.05; hence, there was no significant difference between the temperatures.

Also, the proposed method analysis had a X^2 = 0.201 $< X^2_{0.975,1} = 5.02$; therefore, we did not reject the null hypothesis and conclude the population medians are equal.

The proposed method had the minimum variance of 44.82 compared to the variance of Wilcoxon signed rank test with a variance of 7848.75. Minimum variance showed that the proposed method is much better than the Wilcoxon signed rank test which was better than the Sign test. Standard Error of proposed method was 0.947 while standard error of Wilcoxon signed rank is 13.207. Hence, the proposed method is better than the Wilcoxon signed rank test and the sign test. Finally, the efficiency of the proposed test

statistic relative to the Wilcoxon sign ranked test is 175.12, which was greater than 1, therefore the proposed method is more efficient than the Wilcoxon signed rank test. This implied that the proposed method was also more efficient than the sign test.
The analysis of the three test showed that there was no significant difference between the temperatures at day 1 and day 2. In order to know if Oyeka and Umeh method (Oyeka and Umeh 2012) was indeed more efficient than the sign test and the Wilcoxon signed rank test, the efficiency, variance and the standard error of the tests were compared, which showed that the proposed method by Oyeka and Umeh (Oyeka and Umeh 2012) was better. Instead of taking the difference between the pairs of observation and assigning ranks to the difference, ranks should be assigned first to the pairs of observation before taking the difference (Oyeka and Umeh 2012), which is a better way of ranking data because it used all available information on the data unlike the sign and Wilcoxon signed rank test which ignored the zero difference between the pairs of observation.

CONCLUSION

The proposed method is more efficient than the Wilcoxon signed rank test and the sign test since the efficiency test proves the proposed method to be more powerful. The three tests showed there is no significance difference in the observations and that they are from the same population.The relationship between the three tests is that the observations are ranked and difference between the pairs of observation was taken to know the signs of the difference which was used in the test.

The minimum variance and the standard error tell us that the proposed method is a better statistic since it has the minimum variance and smallest standard error.

The proposed method has an added advantage because it uses all the available information unlike the sign test that uses only the sign and the Wilcoxon signed rank that ignores the tied observations. This research also proves that there is a better way of ranking pairs of observation without losing any information.

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