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AXIAL FORCE INFLUENCE ON THE VIBRATION OF PRESTRESSED PLATE WITH GENERAL BOUNDARY CONDITIONS AND RESTING ON WINKLER FOUNDATION

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ABSTRACT

The influence of Axial Force on the response to moving concentrated masses of prestressed rectangular plates with general boundary conditions and resting on Winkler elastic foundations is investigated in this work. The governing fourth order partial differential equation with variable and singular coefficients is solved using a technique based on separation of variables, a modification of the Struble's technique and method of integral transformations. The numerical results in plotted curves show that the response amplitudes of the plate decrease as the value of the axial force in x-direction (N_x) increases, the axial force in y-direction (N_y) produces the same effect but the influence of N_x is more noticeable than that of N_y for both cases of moving force and moving mass problems of the prestressed rectangular plate resting on Winkler elastic foundation for the classical boundary conditions considered. Also, for fixed values of N_x and N_y, the transverse deflections of the rectangular plates under the actions of moving masses are higher than those when only the force effects of the moving loads are considered, the results also show that the critical speed for the moving mass problem is reached prior to that of the moving force problem which implies that the moving force solution is not a safe approximation to the moving mass problem,; hence, it is risky to rely on a design based on the moving force solution. It is further shown that the response amplitudes of the moving mass problem increase with increasing mass ratio and approach the response amplitudes of the moving force as the mass ratio approaches zero for the prestressed rectangular plates with general boundary conditions and resting on Winkler elastic foundation for all the classical boundary conditions considered.

Key words: Axial force, Prestress, Mass ratio, Resonance, Moving Force, Moving Mass.

INTRODUCTION

Researchers have investigated the problem of structures (such as bridges, roadways and decking slabs) constantly acted upon by moving masses and this problem continues to motivate a variety of investigations (Inglis, 1934; Fryba, 1972; Gbadeyan and Aiyesimi, 1990; Oni, 1991; Gbadeyan and Oni, 1995). In most analytical studies in Engineering and Mathematical Physics, structural members are commonly modeled as a beam or as a plate.

Aside the problem arising from the inclusion of the inertia terms in moving mass problems, difficulties often arise from the type of specified end-conditions. There are four classical boundary conditions that are commonly of practical interest to an applied Mathematician or an Engineer. These are Pinned end conditions (Simply supported end conditions), Fixed / Clamped end conditions, Free end conditions and Sliding end conditions (Fryba, 1972; Jaeger and Strarfield, 1979).

Several researchers have also made tremendous efforts in the study of dynamics of structures under moving loads (Adams, 1995; Savin, 2001; Shadnam *et al*., 2001; Oni and Awodola, 2003; Oni, 2004; Oni and Omolofe, 2005; Omer and Aitung, 2006; Jia-Jang, 2006). In most of the investigations in literature on vibration of rectangular plate under moving loads and resting on elastic foundations, work has been restricted to cases when the plate is not prestressed. The more complicated case, when the plate is prestressed has been neglected, where this is considered, work has been restricted to the simplest form of the problem when the structure is simply supported (Awodola, 2014).

Often, engineers create artificial stresses in structures before loading, such artificial stresses are forces which may act axially or otherwise. When they act axially, they are called axial forces. The artificial stresses are also called prestress. The aim of prestressed structures is to limit tensile stresses and hence flexural cracking or bending in the structure under working conditions. If the structure is subjected to a force parallel to its axes in addition to the lateral loading, the local equilibrium of forces is altered because the axial force interacts with the lateral displacement to produce an additional term (Clough and Penziens, 1975). This additional term due to the axial force increases the complexity of the problem.

In all the aforementioned investigations, considerations have been limited to cases of one-dimensional (beam) problems. Where two-dimensional (plate) problems have been considered, no considerations have been given to the class of dynamical problems in which the plate is prestressed, especially the influence of the prestress (axial force) on such dynamical system with general classical boundary conditions.

This work however decided to investigate the influence of axial force on the response to moving concentrated masses of prestressed rectangular plates with general boundary conditions and resting on Winkler elastic foundations.

GOVERNING EQUATION

The equation governing the dynamic transverse displacement U(x,y,t) of a prestressed rectangular plate when it is resting on a Winkler elastic foundation and traversed by several concentrated masses M_i moving with velocity c_i (issuing from point y = s on the y – axis) is the fourth order partial differential equation given by

$$D\,\nabla^4 U(x,y,t) + \mu \frac{\partial^2 U(x,y,t)}{\partial t^2} = \mu R_0 \left[\frac{\partial^4}{\partial t^2 \partial x^2} + \frac{\partial^4}{\partial t^2 \partial y^2} \right] U(x,y,t) + \left[N_x \frac{\partial^2 U(x,y,t)}{\partial x^2} + N_y \frac{\partial^2 U(x,y,t)}{\partial y^2} \right]$$

$$- F_0 U(x,y,t) + \sum_{i=1}^{N} \left[M_i g \delta(x - c_i t)\delta(y-s) - M_i \left(\frac{\partial^2}{\partial t^2} + 2c_i \frac{\partial^2}{\partial t \partial x} + c_i^2 \frac{\partial^2}{\partial x^2} \right) U(x,y,t)\delta(x-c_i t)\delta(y-s) \right] \quad (1)$$

where $D = \dfrac{Eh^2}{12(1-v)}$ is the bending rigidity of the plate, ∇^2 is the two-dimensional Laplacian operator, h is the plate's thickness, E is the Young's Modulus, v is the Poisson's ratio $(v<1)$, μ is the mass per unit area of the plate, N_x and N_y are the axial forces in x and y directions respectively, R_0 is the Rotatory inertia correction factor, F_0 is the foundation's stiffness, x and y are respectively the spatial coordinates in x and y directions and t is the time coordinate.

The boundary conditions are arbitrary and the initial conditions, without any loss of generality, is taken as

$$U(x,y,t) = 0 = \frac{\partial U(x,y,t)}{\partial t} \quad (2)$$

ANALYTICAL APPROXIMATE SOLUTION

Evidently, an exact closed form solution of the above fourth order partial differential equation (1) does not exist. Consequently, an approximate solution is sought. Thus, the technique based on separation of variable described in Awodola (2014) is employed. This versatile technique requires that the solution of equation (1) takes the form

$$U(x,y,t) = \sum_{n=1}^{\infty} \phi_n(x,y) T_n(t) \quad (3)$$

where ϕ_n are the known eigen functions of the plate with the same boundary conditions and have the form of, Shadnam et al (2001)

$$\nabla^4 \phi_n - \omega_n^4 \phi_n = 0 \tag{4}$$

where

$$\omega_n^4 = \frac{\Omega_n^2 \mu}{D} \tag{5}$$

Ω_n, n = 1, 2, 3, … , are the natural frequencies of the dynamical system and $T_n(t)$ are amplitude functions which have to be calculated.

In order to solve the equation (1), it is rewritten as

$$\frac{D}{\mu}\nabla^4 U(x,y,t) + \frac{\partial^2 U(x,y,t)}{\partial t^2} = R_0\left[\frac{\partial^4}{\partial t^2 \partial x^2} + \frac{\partial^4}{\partial t^2 \partial y^2}\right]U(x,y,t) + \left[N_x \frac{\partial^2 U(x,y,t)}{\partial x^2} + N_y \frac{\partial^2 U(x,y,t)}{\partial y^2}\right]$$
$$-\frac{F_0}{\mu}U(x,y,t) + \sum_{i=1}^{N}\left[\frac{M_i g}{\mu}\delta(x-c_i t)\delta(y-s) - \frac{M_i}{\mu}\left(\frac{\partial^2}{\partial t^2} + 2c_i\frac{\partial^2}{\partial t \partial x} + c_i^2\frac{\partial^2}{\partial x^2}\right)U(x,y,t)\delta(x-c_i t)\delta(y-s)\right] \tag{6}$$

Rewriting the right hand side of equation (6) in the form of a series, we have

$$R_0\left[\frac{\partial^4}{\partial t^2 \partial x^2} + \frac{\partial^4}{\partial t^2 \partial y^2}\right]U(x,y,t) + \left[N_x \frac{\partial^2 U(x,y,t)}{\partial x^2} + N_y \frac{\partial^2 U(x,y,t)}{\partial y^2}\right] - \frac{F_0}{\mu}U(x,y,t)$$
$$+\sum_{i=1}^{N}\left[\frac{M_i g}{\mu}\delta(x-c_i t)\delta(y-s) - \frac{M_i}{\mu}\left(\frac{\partial^2}{\partial t^2} + 2c_i\frac{\partial^2}{\partial t \partial x} + c_i^2\frac{\partial^2}{\partial x^2}\right)U(x,y,t)\delta(x-c_i t)\delta(y-s)\right] = \sum_{n=1}^{\infty}\phi_n(x,y)B_n(t) \tag{7}$$

When equation (3) is used in equation (7) we have

$$\sum_{n=1}^{\infty}\Big\{R_0\left[\phi_{n,xx}(x,y)T_{n,tt}(t) + \phi_{n,yy}(x,y)T_{n,tt}(t)\right] - \frac{F_0}{\mu}\phi_n(x,y)T_n(t) + \frac{N_x}{\mu}\phi_{n,xx}(x,y)T_n(t)$$
$$+\frac{N_y}{\mu}\phi_{n,yy}(x,y)T_n(t) + \sum_{i=1}^{N}\Big[\ \frac{M_i g}{\mu}\delta(x-c_i t)\delta(y-s) - \frac{M_i}{\mu}\Big(\phi_n(x,y)T_{n,tt}(t) + 2c_i\phi_{n,x}(x,y)T_{n,t}(t)$$
$$+c_i^2\phi_{n,xx}(x,y)T_n(t)\ \Big)\delta(x-c_i t)\delta(y-s)\ \Big]\ \Big\} = \sum_{n=1}^{\infty}\phi_n(x,y)B_n(t) \tag{8}$$

Multiplying both sides of equation (8) by $\phi_p(x,y)$ and integrating on area A of the plate, we have

$$\sum_{n=1}^{\infty}\int_A \Big\{R_0\big[\phi_{n,xx}(x,y)\phi_p(x,y)T_{n,tt}(t)+\phi_{n,yy}(x,y)\phi_p(x,y)T_{n,tt}(t)\big]+\frac{N_x}{\mu}\phi_{n,xx}(x,y)\phi_p(x,y)T_n(t)$$
$$+\frac{N_y}{\mu}\phi_{n,yy}(x,y)\phi_p(x,y)T_n(t)-\frac{F_0}{\mu}\phi_n(x,y)\phi_p(x,y)T_n(t)+\sum_{i=1}^{N}\Big[\ \frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_i t)\delta(y-s)$$
$$-\frac{M_i}{\mu}\Big(\phi_n(x,y)\phi_p(x,y)T_{n,tt}(t)+2c_i\phi_{n,x}(x,y)\phi_p(x,y)T_{n,t}(t)+c_i^2\phi_{n,xx}(x,y)\phi_p(x,y)T_n(t)\ \Big)\delta(x-c_i t)\delta(y-s)\ \Big]\Big\}dA$$
$$=\sum_{n=1}^{\infty}\int_A \phi_n(x,y)\phi_p(x,y)B_n(t)\,dA \qquad (9)$$

Considering the orthogonality of $\phi_n(x,y)$, we have

$$B_n(t)=\frac{1}{P^*}\sum_{n=1}^{\infty}\int_A \Big\{R_0\big[\phi_{n,xx}(x,y)\phi_p(x,y)T_{n,tt}(t)+\phi_{n,yy}(x,y)\phi_p(x,y)T_{n,tt}(t)\big]+\frac{N_x}{\mu}\phi_{n,xx}(x,y)T_n(t)\phi_p(x,y)$$
$$+\frac{N_y}{\mu}\phi_{n,yy}(x,y)\phi_p(x,y)T_n(t)-\frac{F_0}{\mu}\phi_n(x,y)\phi_p(x,y)T_n(t)+\sum_{i=1}^{N}\Big[\ \frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_i t)\delta(y-s)$$
$$-\frac{M_i}{\mu}\Big(\phi_n(x,y)\phi_p(x,y)T_{n,tt}(t)+2c_i\phi_{n,x}(x,y)\phi_p(x,y)T_{n,t}(t)+c_i^2\phi_{n,xx}(x,y)\phi_p(x,y)T_n(t)\ \Big)\delta(x-c_i t)\delta(y-s)\ \Big]\Big\}dA \qquad (10)$$

where $P^*=\int_A \phi_p^2\,dA$

Using (10) and taking into account (4) and (5), equation (6) can be written as

$$\phi_n(x,y)\left[\frac{D\omega_n^4}{\mu}T_n(t)+T_{n,tt}(t)\right]=\frac{\phi_n(x,y)}{P^*}\sum_{q=1}^{\infty}\int_A \Big\{R_0\big[\phi_{q,xx}(x,y)\phi_p(x,y)T_{q,tt}(t)+\phi_{q,yy}(x,y)\phi_p(x,y)T_{q,tt}(t)\big]$$
$$-\frac{F_0}{\mu}\phi_q(x,y)\phi_p(x,y)T_q(t)+\frac{N_x}{\mu}\phi_{q,xx}(x,y)\phi_p(x,y)T_q(t)+\frac{N_y}{\mu}\phi_{q,yy}(x,y)\phi_p(x,y)T_q(t)$$
$$+\sum_{i=1}^{N}\Big[\ \frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_i t)\delta(y-s)-\frac{M_i}{\mu}\Big(\phi_q(x,y)\phi_p(x,y)T_{q,tt}(t)$$
$$+2c_i\phi_{q,x}(x,y)\phi_p(x,y)T_{q,t}(t)+c_i^2\phi_{q,xx}(x,y)\phi_p(x,y)T_q(t)\ \Big)\delta(x-c_i t)\delta(y-s)\ \Big]\Big\}dA \qquad (11)$$

Equation (11) implies that

$$T_{n,tt}(t)+\frac{D\omega_n^4}{\mu}T_n(t)=\frac{1}{P^*}\sum_{q=1}^{\infty}\int_A \Big\{R_0\left[\phi_{q,xx}(x,y)\phi_p(x,y)T_{q,tt}(t)+\phi_{q,yy}(x,y)\phi_p(x,y)T_{q,tt}(t)\right]$$
$$+\frac{N_x}{\mu}\phi_{q,xx}(x,y)\phi_p(x,y)T_q(t)+\frac{N_y}{\mu}\phi_{q,yy}(x,y)\phi_p(x,y)T_q(t)-\frac{F_0}{\mu}\phi_q(x,y)\phi_p(x,y)T_q(t)$$
$$+\sum_{i=1}^{N}\Big[\ \frac{M_i g}{\mu}\phi_p(x,y)\delta(x-c_it)\delta(y-s)-\frac{M_i}{\mu}\Big(\phi_q(x,y)\phi_p(x,y)T_{q,tt}(t)$$
$$+2c_i\phi_{q,x}(x,y)\phi_p(x,y)T_{q,t}(t)+c_i^2\phi_{q,xx}(x,y)\phi_p(x,y)T_q(t)\ \Big)\delta(x-c_it)\delta(y-s)\ \Big]\Big\}dA \qquad (12)$$

Equation (12) is a set of coupled second order ordinary differential equations.
Expressing the Dirac-Delta function in the Fourier cosine series as Shadnam *et al*. (2001)

$$\delta(x-c_it)=\frac{1}{L_X}\left[1+2\sum_{j=1}^{\infty}\cos\frac{j\pi c_it}{L_X}\cos\frac{j\pi x}{L_X}\right] \text{ and } \delta(y-s)=\frac{1}{L_Y}\left[1+2\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}\cos\frac{k\pi y}{L_Y}\right] \qquad (13)$$

equation (12) then becomes

$$\frac{d^2T_n(t)}{dt^2}+\alpha_n^2T_n(t)-\frac{1}{P^*}\sum_{q=1}^{\infty}\Bigg\{R_0P_1^*\frac{d^2T_q(t)}{dt^2}-\frac{1}{\mu}\left(F_0P_2^*-N_xk^0-N_yk^1\right)T_q(t)$$
$$-\sum_{i=1}^{N}\ \frac{M_i}{L_XL_Y\mu}\Bigg[2\Bigg(\frac{P_3^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi c_it}{L_X}P_3^{***}(j)$$
$$+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi c_it}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\Bigg)\frac{d^2T_q(t)}{dt^2}+4c_i\Bigg(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi c_it}{L_X}P_4^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi c_it}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\Bigg)\frac{dT_q(t)}{dt}$$
$$+2c_i^2\Bigg(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi c_it}{L_X}P_5^{***}(j)$$
$$+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi c_it}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\Bigg)T_q(t)\Bigg]\Bigg\}=\sum_{i=1}^{N}\frac{M_ig}{P^*\mu}\phi_p(c_it,s) \qquad (14)$$

where $\alpha_n^2=\frac{D\omega_n^4}{\mu}$, $k^0=\int_0^{L_X}\int_0^{L_Y}\ \phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx$, $k^1=\int_0^{L_X}\int_0^{L_Y}\ \phi_{n,yy}(x,y)\phi_p(x,y)\,dy\,dx$,

$P_1^*=k^0+k^1$, $P_2^*=\int_0^{L_X}\int_0^{L_Y}\ \phi_n(x,y)\phi_p(x,y)\,dy\,dx$, $P_3^*=\int_0^{L_X}\int_0^{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx$,

$P_3^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx$, $P_3^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_n(x,y)\phi_p(x,y)\,dy\,dx$,

$P_3^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx$, $P_4^*=\int_0^{L_X}\int_0^{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx$,

$P_4^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx$, $P_4^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx$,

$$P_4^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,\;\; P_5^{*}=\int_0^{L_X}\int_0^{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,\;\; P_5^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx$$

$$and\quad P_5^{****}(j,k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

Equation (14) is the transformed equation governing the problem of the rectangular plate on a Winkler elastic foundation.

In what follows, $\phi_n(x,y)$ are assumed to be the products of the beam functions $\psi_{ni}(x)$ and $\psi_{nj}(y)$, Lee and Ng (1996). That is

$$\phi_n(x,y)=\psi_{ni}(x)\psi_{nj}(y) \tag{15}$$

These beam functions can be defined respectively, as

$$\psi_{ni}(x)=\sin\frac{\Omega_{ni}x}{L_X}+A_{ni}\cos\frac{\Omega_{ni}x}{L_X}+B_{ni}\sinh\frac{\Omega_{ni}x}{L_X}+C_{ni}\cosh\frac{\Omega_{ni}x}{L_X} \tag{16}$$

and
$$\psi_{nj}(y)=\sin\frac{\Omega_{nj}y}{L_Y}+A_{nj}\cos\frac{\Omega_{nj}y}{L_Y}+B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y}+C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y} \tag{17}$$

where A_{ni}, A_{nj}, B_{ni}, B_{nj}, C_{ni} and C_{nj} are constants determined by the boundary conditions. Ω_{ni} and Ω_{nj} are called the mode frequencies.
Thus for the single mass M with velocity c equation (14) reduces to

$$\frac{d^2T_n(t)}{dt^2}+\alpha_n^2T_n(t)-\frac{1}{P^*}\sum_{q=1}^{\infty}\Bigg\{R_0P_1^*\frac{d^2T_q(t)}{dt^2}-\frac{1}{\mu}\left(F_0P_2^*-N_xk^0-N_yk^1\right)T_q(t)-\Gamma\Bigg[2\Bigg(\frac{P_3^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\Bigg)\frac{d^2T_q(t)}{dt^2}+4c\Bigg(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\Bigg)\frac{dT_q(t)}{dt}+2c^2\Bigg(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\Bigg)T_q(t)\Bigg]\Bigg\}=\frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \tag{18}$$

where $\Gamma=\dfrac{M}{L_XL_Y\mu}$

Equation (18) is the fundamental equation of our problem when the rectangular plate has arbitrary end support conditions. In what follows, we shall discuss two special cases of the equation (18) namely; the moving force and the moving mass problems.

CASE I: MOVING FORCE PROBLEM

By setting Γ = 0 in equation (18), an approximate model of the differential equation describing the response of a rectangular plate resting on a Winkler elastic foundation and traversed by a moving force would be obtained.

Thus, setting Γ = 0 in equation (18), we have

$$\frac{d^2T_n(t)}{dt^2} + \alpha_n^2 T_n(t) - \frac{P_1^* R_0}{P^*}\sum_{q=1}^{\infty}\frac{d^2T_q(t)}{dt^2} + \frac{1}{\mu P^*}\left(F_0 P_2^* - N_x k^0 - N_y k^1\right)\sum_{q=1}^{\infty} T_q(t) = \frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \quad (19)$$

Evidently, an exact analytical solution to this equation is not possible. Consequently, the approximate analytical solution technique, which is a modification of the asymptotic method of Struble shall be used (Awodola and Oni, 2013).

To solve equation (19), first, we neglect the rotatory inertial term and rearrange the equation to take the form

$$\frac{d^2T_n(t)}{dt^2} + \left[\alpha_n^2 + \Gamma^* P_2^*\right] T_n(t) + \Gamma^* P_2^* \sum_{\substack{q=1 \\ q\neq n}}^{\infty} T_q(t) = \frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \quad (20)$$

where

$$\Gamma^* = \frac{1}{\mu P^*}\left(F_0 - N_x \frac{k^0}{P_2^*} - N_y \frac{k^1}{P_2^*}\right) \quad (21)$$

Consider a parameter λ < 1 for any arbitrary ratio Γ^* defined as

$$\lambda = \frac{\Gamma^*}{1+\Gamma^*} \quad (22)$$

so that

$$\Gamma^* = \lambda + o(\lambda^2) \quad (23)$$

Thus, Using struble's technique (Awodola and Oni, 2013), the homogeneous part of equation (20) can be replaced with

$$\frac{d^2T_n(t)}{dt^2} + \gamma_s^2 T_n(t) = 0 \quad (24)$$

where

$$\gamma_s = \alpha_n + \frac{\lambda P_2^*}{2\alpha_n} \quad (25)$$

represents the modified frequency due to the effects of the axial forces N_x and N_y.
Using equation (25), equation (19) can be written as

$$\frac{d^2T_n(t)}{dt^2} + \gamma_s^2 T_n(t) - \frac{P_1^* R_0}{P^*}\sum_{q=1}^{\infty}\frac{d^2T_q(t)}{dt^2} = \frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \quad (26)$$

The homogenous part of equation (26) is rearranged to take the form

$$\frac{d^2T_n(t)}{dt^2}+\frac{\gamma_s^2}{1-\lambda_0 P_1^*}T_n(t)-\frac{\lambda_0 P_1^*}{1-\lambda_0 P_1^*}\sum_{\substack{q=1\\q\neq n}}^{\infty}\frac{d^2T_q(t)}{dt^2}=0 \qquad (27)$$

where $\lambda_0=\frac{R_0}{P^*}$

Now, consider the parameter $\varepsilon_0 < 1$ for any arbitrary mass ratio λ_0 defined as

$$\varepsilon_0=\frac{\lambda_0}{1+\lambda_0} \qquad (28)$$

It can be shown that

$$\lambda_0=\varepsilon_0+o(\varepsilon_0^2) \qquad (29)$$

Following the same argument, equation (27) can be replaced with

$$\frac{d^2T_n(t)}{dt^2}+\gamma_{sf}^2T_n(t)=0 \qquad (30)$$

where

$$\gamma_{sf}=\gamma_s\left[1+\frac{\varepsilon_0 P_1^*}{2}\right] \qquad (31)$$

is the modified frequency corresponding to the frequency of the free system due to the presence of the rotatory inertia correction factor. It is observed that when $\varepsilon_0 = 0$, we recover the frequency of the moving force problem when the rotatory inertia effect is neglected.

In order to solve the non-homogenous equation (26), the differential operator which acts on $T_n(t)$ is replaced by the equivalent free system operator defined by the modified frequency γ_{sf}. Thus

$$\frac{d^2T_n(t)}{dt^2}+\gamma_{sf}^2T_n(t)=K_0\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (32)$$

where $K_0=\frac{Mg}{P^*\mu}$ (33)

Therefore, the moving force problem is reduced to the non-homogeneous ordinary differential equation given as

$$\frac{d^2T_n(t)}{dt^2}+\gamma_{sf}^2T_n(t)=K_0\Psi_{pj}(s)\left[\sin\alpha_{pi}t+A_{pi}\cos\alpha_{pi}t+B_{pi}\sinh\alpha_{pi}t+C_{pi}\cosh\alpha_{pi}t\right] \qquad (34)$$

where $\alpha_{pi}=\frac{\Omega_{pi}c}{L_X}$

When equation (34) is solved in conjunction with the initial conditions (2), one obtains expression for $T_n(t)$. Thus in view of equation (3), one obtains

$$U(x,y,t)=\sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty}\frac{K_0\Psi_{pj}(s)}{\gamma_{sf}[\gamma_{sf}^4-\alpha_{pi}^4]}\{[\gamma_{sf}^2-\alpha_{pi}^2][C_{pi}\gamma_{sf}(\cosh\alpha_{pi}t-\cos\gamma_{sf}t)+B_{pi}(\gamma_{sf}\sinh\alpha_{pi}t-\alpha_{pi}\sin\gamma_{sf}t)]$$
$$+[\gamma_{sf}^2+\alpha_{pi}^2][A_{pi}\gamma_{sf}(\cos\alpha_{pi}t-\cos\gamma_{sf}t)-(\alpha_{pi}\sin\gamma_{sf}t-\gamma_{sf}\sin\alpha_{pi}t)]\}[\sin\frac{\Omega_{ni}x}{L_X}+A_{ni}\cos\frac{\Omega_{ni}x}{L_X}+B_{ni}\sinh\frac{\Omega_{ni}x}{L_X}$$
$$+C_{ni}\cosh\frac{\Omega_{ni}x}{L_X}][\sin\frac{\Omega_{nj}y}{L_Y}+A_{nj}\cos\frac{\Omega_{nj}y}{L_Y}+B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y}+C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y}] \qquad (35)$$

Equation (35) represents the transverse displacement response to a moving force of a rectangular plate resting on Winkler elastic foundation.

CASE II: MOVING MASS PROBLEM

If the mass of the moving load is commensurable with that of the structure, the inertia effect of the moving mass is not negligible. Thus $\Gamma \neq 0$ and one is required to solve the entire equation (18) when no term of the coupled differential equation is neglected. This is termed the moving mass problem. Thus, equation (18) can be rewritten in the form

$$\left[1+\frac{2\varepsilon}{P^*}\left(\frac{P_2^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\right]\frac{d^2T_n(t)}{dt^2}$$
$$+\frac{4\varepsilon c}{P^*}\left(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dT_n(t)}{dt}$$
$$+\left[\gamma_{sf}^2+\frac{2\varepsilon c^2}{P^*}\left(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)\right]T_n(t)$$
$$+\frac{\varepsilon}{P^*}\sum_{\substack{q=1\\q\neq n}}\left[2\left(\frac{P_2^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\frac{d^2T_q(t)}{dt^2}\right.$$
$$4c\left(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dT_q(t)}{dt}$$
$$\left.+2c^2\left(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)T_q(t)\right]$$
$$=\frac{\varepsilon gL_XL_Y}{P^*}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (36)$$

where $\varepsilon=\frac{M}{L_XL_Y\mu}$

We rearrange equation (36) to take the form

$$\frac{d^2T_n(t)}{dt^2}+\frac{\mu_0R_2(t)}{1+\mu_0R_1(t)}\frac{dT_n(t)}{dt}+\frac{\gamma_{sf}^2+\mu_0R_3(t)}{1+\mu_0R_1(t)}T_n(t)+\frac{\mu_0}{1+\mu_0R_1(t)}\sum_{\substack{q=1\\q\neq n}}^{\infty}\Big[R_1(t)\frac{d^2T_q(t)}{dt^2}+R_2(t)\frac{dT_q(t)}{dt}$$
$$+R_3(t)T_q(t)\Big]=\frac{\mu_0gL_XL_Y}{[1+\mu_0R_1(t)]P^*}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (37)$$

where ε has been written as a function of the mass ratio μ_o,

$$R_1(t) = \frac{2}{P^*}\left[\frac{P_2^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right]$$

$$R_2(t) = \frac{2c}{P^*}\left[\frac{P_4^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right]$$

$$R_3(t) = \frac{2c^2}{P^*}\left[\frac{P_5^*}{2} + \sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k) + \sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j) + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right]$$

Considering the homogeneous part of the equation (36) and going through the same arguments and analysis as in the previous case, the modified frequency corresponding to the frequency of the free system due to the presence of the moving mass is

$$\beta_{sf} = \gamma_{sf}\left[1 - \frac{\mu_0}{2}\left(R_1 - \frac{R_3}{\gamma_{sf}^2}\right)\right] \tag{38}$$

retaining terms to $o(\mu_o)$ only.

Thus, to solve the non-homogeneous equation (36), the differential operator which acts on $T_n(t)$ and $T_q(t)$ is replaced by the equivalent free system operator defined by the modified frequency β_{sf}. Therefore, taking into account equations (16) and (17), we have

$$\frac{d^2T_n(t)}{dt^2} + \beta_{sf}^2 T_n(t) = G_0\Psi_{pj}(s)(\sin\alpha_{pi}t + A_{pi}\cos\alpha_{pi}t + B_{pi}\sinh\alpha_{pi}t + C_{pi}\cosh\alpha_{pi}t) \tag{39}$$

where $G_0 = \dfrac{\mu_0 g L_X L_Y}{P^*}$ (40)

Therefore, when equation (39) is solved in conjunction with the initial conditions, one obtains expression for $T_n(t)$ and in view of equation (3), one obtains

$$U(x,y,t) = \sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty}\frac{G_0\Psi_{pj}(s)}{\beta_{sf}[\beta_{sf}^4 - \alpha_{pi}^4]}\{[\beta_{sf}^2 - \alpha_{pi}^2][C_{pi}\beta_{sf}(\cosh\alpha_{pi}t - \cos\beta_{sf}t) + B_{pi}(\beta_{sf}\sinh\alpha_{pi}t - \alpha_{pi}\sin\beta_{sf}t)]$$
$$+ [\beta_{sf}^2 + \alpha_{pi}^2][A_{pi}\beta_{sf}(\cos\alpha_{pi}t - \cos\beta_{sf}t) - (\alpha_{pi}\sin\beta_{sf}t - \beta_{sf}\sin\alpha_{pi}t)]\}[\sin\frac{\Omega_{ni}x}{L_X} + A_{ni}\cos\frac{\Omega_{ni}x}{L_X} + B_{ni}\sinh\frac{\Omega_{ni}x}{L_X}$$
$$+ C_{ni}\cosh\frac{\Omega_{ni}x}{L_X}][\sin\frac{\Omega_{nj}y}{L_Y} + A_{nj}\cos\frac{\Omega_{nj}y}{L_Y} + B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y} + C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y}] \tag{41}$$

Equation (41) is the transverse displacement response to a moving mass of a rectangular plate resting on Winkler elastic foundation. The constants A_{ni}, A_{pi}, A_{nj}, A_{pj}, B_{ni}, B_{pi}, B_{nj}, B_{pj}, C_{ni}, C_{pi}, C_{nj} and C_{pj} are to be determined from the choice of the end support condition.

ANALYSIS OF THE SOLUTION

Next, the phenomenon of resonance is examined. Equation (35) clearly shows that the rectangular plate on a variable Winkler elastic foundation and traversed by a moving force reaches a state of resonance whenever

$$\gamma_{sf} = \frac{\Omega_{pi} c}{L_X} \tag{42}$$

while equation (41) shows that the same plate under the action of a moving mass experiences resonance effect whenever

$$\beta_{sf} = \frac{\Omega_{pi} c}{L_X} \tag{43}$$

where $$\beta_{sf} = \gamma_{sf}\left[1 - \frac{\mu_0}{2}\left(R_1 - \frac{R_3}{\gamma_{sf}^2}\right)\right] \tag{44}$$

Equations (43) and (44) imply that

$$\beta_{sf} = \gamma_{sf}\left[1 - \frac{\mu_0}{2}\left(R_1 - \frac{R_3}{\gamma_{sf}^2}\right)\right] = \frac{\Omega_{pi} c}{L_X} \tag{45}$$

Consequently from equations (42) and (45), for the same natural frequency of the plate, the resonance is reached earlier when we consider the moving mass system than when we consider the moving force system.

ILLUSTRATIVE EXAMPLES

In this section, we shall illustrate the foregoing analysis by two practical examples. Particularly we shall consider classical boundary conditions such as clamped-clamped end conditions and simple-clamped end conditions.

RECTANGULAR PLATE CLAMPED AT ALL EDGES (CLAMPED-CLAMPED)

For a rectangular plate clamped at all its edges, the boundary conditions are given by

$$U(0, y, t) = 0, \quad U(L_X, y, t) = 0, \quad U(x, 0, t) = 0, \quad U(x, L_Y, t) = 0 \tag{46}$$

$$\frac{\partial U(0, y, t)}{\partial x} = 0, \quad \frac{\partial U(L_X, y, t)}{\partial x} = 0, \quad \frac{\partial U(x, 0, t)}{\partial y} = 0, \quad \frac{\partial U(x, L_Y, t)}{\partial y} = 0 \tag{47}$$

Thus for the normal modes

$$\Psi_{ni}(0) = 0, \quad \Psi_{ni}(L_X) = 0, \quad \Psi_{nj}(0) = 0, \quad \Psi_{nj}(L_Y) = 0 \tag{48}$$

$$\frac{\partial \Psi_{ni}(0)}{\partial x} = 0, \quad \frac{\partial \Psi_{ni}(L_X)}{\partial x} = 0, \quad \frac{\partial \Psi_{nj}(0)}{\partial y} = 0, \quad \frac{\partial \Psi_{nj}(L_Y)}{\partial y} = 0 \tag{49}$$

For simplicity, our initial conditions are of the form

$$U(x, y, 0) = 0 = \frac{\partial U(x, y, 0)}{\partial t} \tag{50}$$

Using the boundary conditions and the initial conditions it can be shown that

$$A_{ni} = \frac{\sinh\Omega_{ni} - \sin\Omega_{ni}}{\cos\Omega_{ni} - \cosh\Omega_{ni}} = \frac{\cos\Omega_{ni} - \cosh\Omega_{ni}}{\sin\Omega_{ni} + \sinh\Omega_{ni}}, \quad \Rightarrow \quad A_{pi} = \frac{\sinh\Omega_{pi} - \sin\Omega_{pi}}{\cos\Omega_{pi} - \cosh\Omega_{pi}}$$
$$A_{nj} = \frac{\sinh\Omega_{nj} - \sin\Omega_{nj}}{\cos\Omega_{nj} - \cosh\Omega_{nj}} = \frac{\cos\Omega_{nj} - \cosh\Omega_{nj}}{\sin\Omega_{nj} + \sinh\Omega_{nj}}, \quad \Rightarrow \quad A_{pj} = \frac{\sinh\Omega_{pj} - \sin\Omega_{pj}}{\cos\Omega_{pj} - \cosh\Omega_{pj}} \quad (51)$$

$$B_{ni} = -1, \Rightarrow B_{pi} = -1 \qquad C_{ni} = -A_{ni} \Rightarrow C_{pi} = -A_{pi}$$
$$B_{nj} = -1 \Rightarrow B_{pj} = -1, \qquad C_{nj} = -A_{nj} \Rightarrow C_{pj} = -A_{pj} \quad (52)$$

and from (58), one obtains $\cos\Omega_{ni}\cosh\Omega_{ni} = 1$ (53)
which is termed the frequency equation for the dynamical problem, such that, Oni and Awodola (2010)

$$\Omega_1 = 4.73004,\ \Omega_2 = 7.85320,\ \Omega_3 = 10.99561 \quad (54)$$

Using (51), (52) and (54) in equations (35) and (41) one obtains the displacement response respectively to a moving force and a moving mass of a rectangular plate resting on a Winkler elastic foundation and clamped at all its edges.

RECTANGULAR PLATE SIMPLY SUPPORTED AT EDGES x = 0, x = L_X AND CLAMPED AT EDGES y = 0, y = L_Y (SIMPLE-CLAMPED)

For a rectangular plate clamped at edges y = 0, y = L_Y with simple supports at edges x = 0, x = L_X, the boundary conditions at such opposite edges are

$$U(0,y,t) = 0, \quad U(L_X,y,t) = 0, \quad U(x,0,t) = 0, \quad U(x,L_Y,t) = 0 \quad (55)$$
$$\frac{\partial^2 U(0,y,t)}{\partial x^2} = 0, \quad \frac{\partial^2 U(L_X,y,t)}{\partial x^2} = 0, \quad \frac{\partial U(x,0,t)}{\partial y} = 0, \quad \frac{\partial U(x,L_Y,t)}{\partial y} = 0 \quad (56)$$

and for the normal modes

$$\Psi_{ni}(0) = 0, \quad \Psi_{ni}(L_X) = 0, \quad \Psi_{nj}(0) = 0, \quad \Psi_{nj}(L_Y) = 0 \quad (57)$$
$$\frac{\partial^2 \Psi_{ni}(0)}{\partial x^2} = 0, \quad \frac{\partial^2 \Psi_{ni}(L_X)}{\partial x^2} = 0, \quad \frac{\partial \Psi_{nj}(0)}{\partial y} = 0, \quad \frac{\partial \Psi_{nj}(L_Y)}{\partial y} = 0 \quad (58)$$

Using the boundary conditions, for the clamped edges, one obtains.

$$A_{nj} = \frac{\sinh\Omega_{nj} - \sin\Omega_{nj}}{\cos\Omega_{nj} - \cosh\Omega_{nj}}, \quad \Rightarrow \quad A_{pj} = \frac{\sinh\Omega_{pj} - \sin\Omega_{pj}}{\cos\Omega_{pj} - \cosh\Omega_{pj}} \quad (59)$$

$$B_{nj} = -1 \Rightarrow B_{pj} = -1,\ C_{nj} = -A_{nj} \Rightarrow C_{pj} = -A_{pj} \quad (60)$$

The frequency equation of the clamped edges is given by the following determinant equation

$$\begin{vmatrix} (\sinh\Omega_{nj} - \sin\Omega_{nj}) & (\cos\Omega_{nj} - \cosh\Omega_{nj}) \\ (\cos\Omega_{nj} - \cosh\Omega_{nj}) & (\sin\Omega_{nj} + \sinh\Omega_{nj}) \end{vmatrix} = 0 \quad (61)$$

which when simplified yields $\cos\Omega_{nj}\cosh\Omega_{nj} = 1$ (62)

For the simple edges, it is readily shown that $A_{ni} = 0$, $B_{ni} = 0$, $C_{ni} = 0$, and $\Omega_{ni} = n_i\pi$

Similarly, $A_{pi} = 0$, $B_{pi} = 0$, $C_{pi} = 0$, and $\Omega_{pi} = p_i\pi$ (63)

Using (59), (60), (62) and (63) in equations (35) and (41) one obtains the displacement response respectively to a moving force and a moving mass of a simple-clamped rectangular plate resting on a Winkler elastic foundation.

NUMERICAL CALCULATIONS AND DISCUSSION OF RESULTS

For the calculations of practical interests in dynamics of structures, a rectangular plate of length L_Y = 0.914m and breadth L_X = 0.457m is considered. The velocity of the mass is assumed to be 0.8123m/s and the values for E, S and Γ are chosen to be $3.109\text{x}10^9\text{kg/m}^2$, 0.4m and 0.2 respectively.

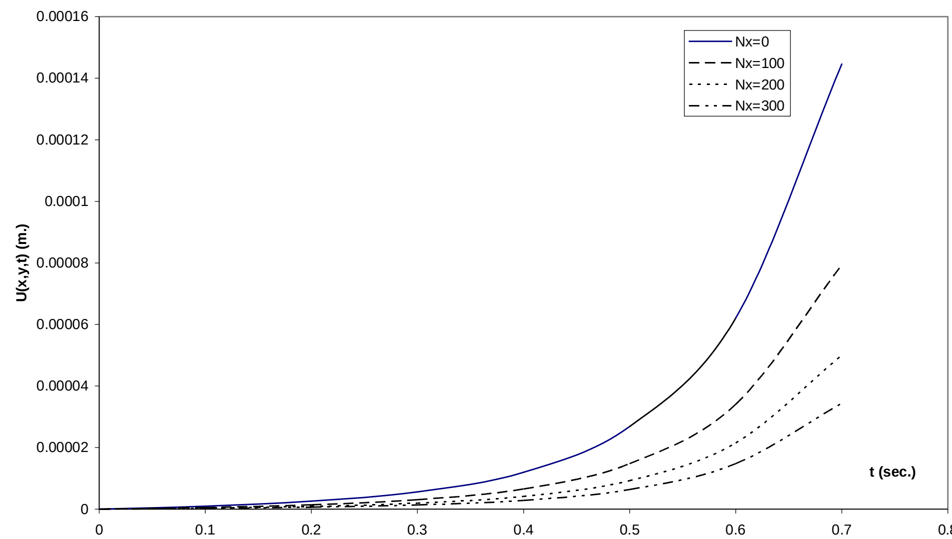

Fig.6.1: Displacement profile of clamped-clamped plate on Winkler elastic foundation and traversed by moving mass for various values of axial force Nx.

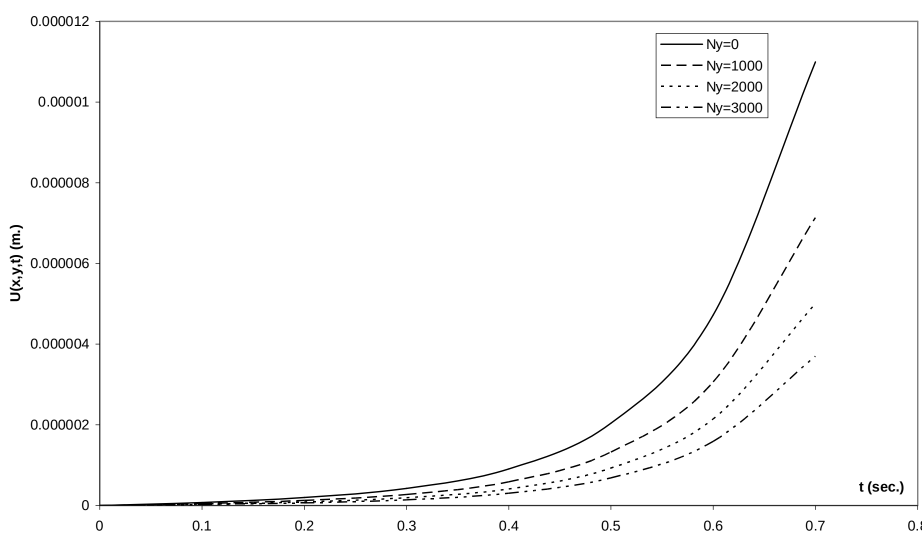

Fig.6.2: Displacement profile of clamped-clamped plate on Winkler elastic foundation and traversed by moving mass for various values of axial force Ny.

Figures 6.1 and 6.2 display the effect of axial forces N_x and N_y respectively on the transverse deflection of the clamped-clamped rectangular plate for the case of moving mass. The graphs show that the response amplitudes decrease as the values of N_x and N_y increase.

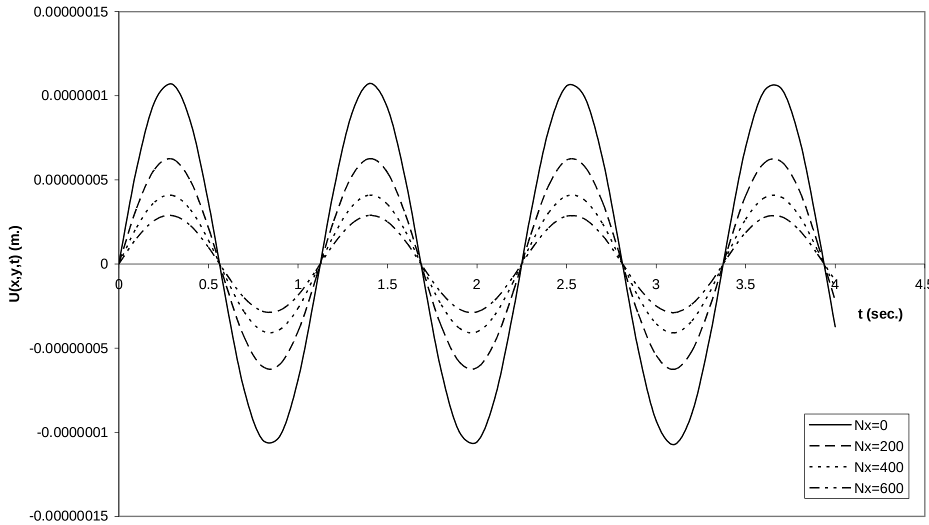

Fig.6.3: Deflection of simple-clamped plate on Winkler elastic foundation and traversed by moving mass for various values of axial force Nx.

The effects of N_x and N_y on the transverse deflection in the case of moving mass displayed in Figures 6.3 and 6.4 respectively show that an increase in the value of each of N_x and N_y decreases the deflection of the simple-clamped rectangular plate resting on Winkler elastic foundation.

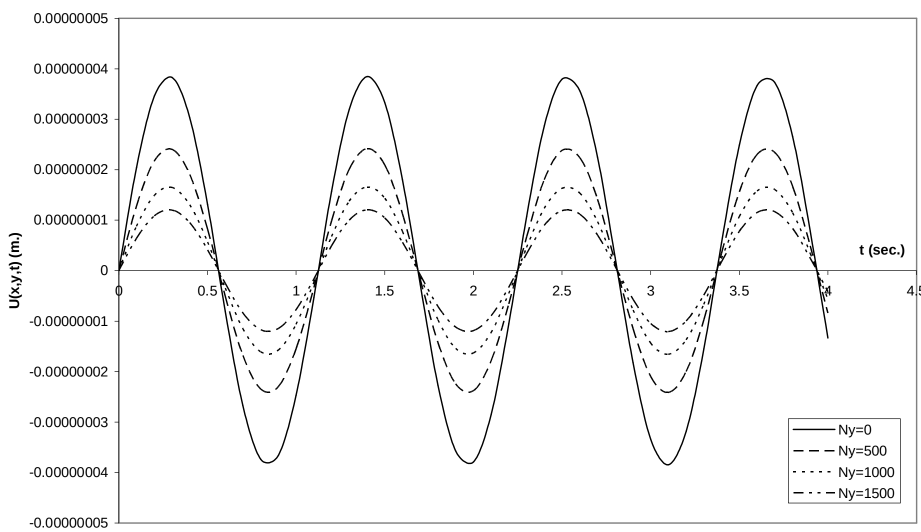

Fig.6.4: Deflection of simple-clamped plate on Winkler elastic foundation and traversed by moving mass for various values of axial force Ny.

Figure 6.5 compares the displacement curves of the moving force and moving mass for a simple-clamped rectangular plate for fixed values of N_x and N_y, the response amplitude of a moving mass is greater than that of a moving force.

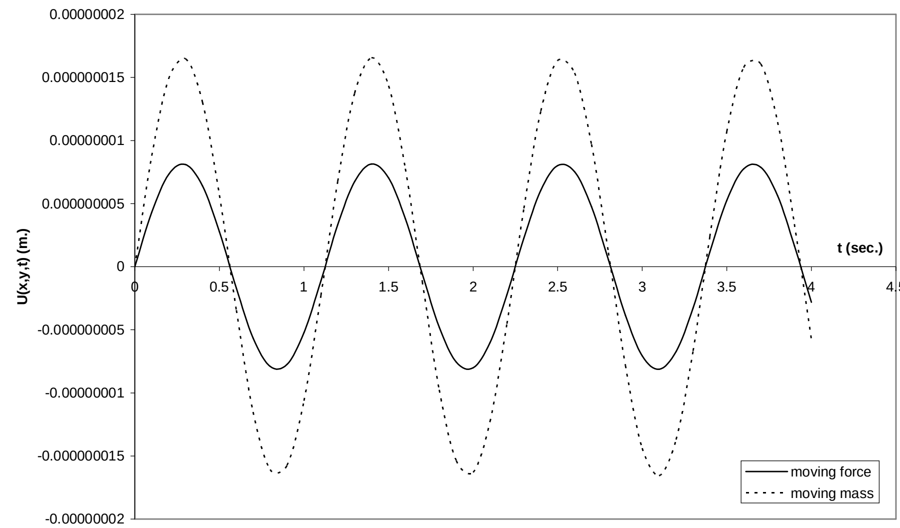

Fig.6.5: Comparison of moving force and moving mass cases of simple-clamped rectangular plate resting on Winkler elastic foundation.

Figure 6.6 displays the effect of the mass ratio Γ on the transverse displacement curves of the moving mass for a simple-clamped rectangular plate for fixed values of N_x and N_y, the response amplitude increases as the mass ratio increases and approaches the deflection of moving force as the mass ratio approaches zero.

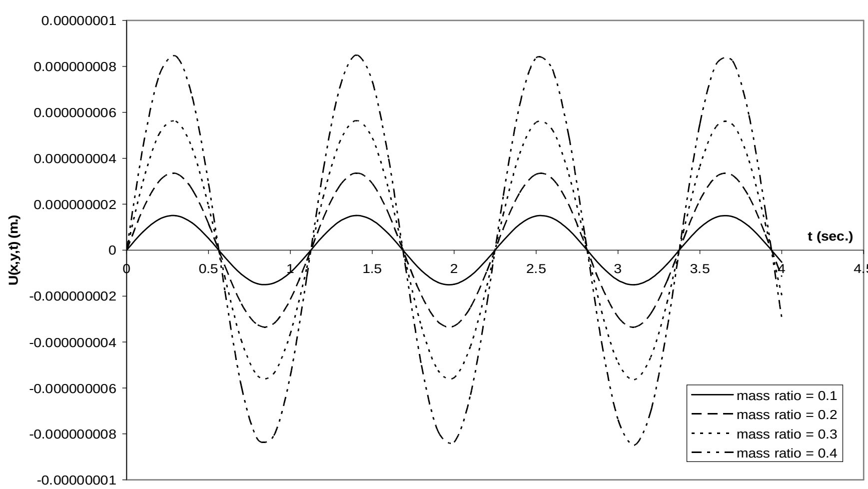

Fig. 6.6: Displacement profile of moving mass of simple-clamped rectangular plate resting on Winkler elastic foundation for various values of mass ratio.

These results hold for other choices of classical boundary conditions.

CONCLUSION

The influence of axial force and mass ratio on the dynamic response to moving masses of rectangular plates with general classical boundary conditions and resting on constant Winkler elastic foundation was considered in this work. The fourth order partial differential equation governing the system was a non-homogenous equation with variable and singular coefficients. The method based on Separation of variables was used to transform the governing equation to a set of coupled second order ordinary differential equations. The modified Struble's technique and the method of integral transformations were employed to obtain the closed form solution of the transformed equation for both cases of moving force and moving mass problems. From the analyses of the solutions, for the same natural frequency, the critical speed (and the natural frequency) for the system of rectangular plate traversed by a moving mass was smaller than that of the same system traversed by a moving force. Thus, for the same natural frequency of the plate, the resonance was reached earlier when we considered the moving mass system than when we considered the moving force system. The analyses showed that the moving force solution was not an upper bound for the accurate solution of the moving mass problem and that as each of the axial forces N_x and N_y increases, the response amplitudes of the plates decrease for both cases of moving force and moving mass problems for all the classical boundary conditions considered. Meanwhile, the effect of N_x was more pronounced than that of N_y Furthermore, the response amplitude for the moving mass problem was greater than that of the moving force problem for fixed values of N_x and N_y, this implied that resonance was reached earlier in moving mass problem than in moving force problem, it was therefore unsafe to rely on the moving force solutions. Also, as the mass ratio Γ approaches zero, the response amplitude of the moving mass problem approached that of the moving force problem of the rectangular plate resting on constant Winkler elastic foundation for the classical boundary conditions considered.

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