Factorial models for determining the actual energy required in ore comminution with ProMax2100 application

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ABSTRACT: Reducing cost in the mineral concentration plant is said to be primarily dependent on reduction of comminution cost especially grinding cost since comminution is the most cost-intensive unit process in the mineral processing plant. In turn, it is known that in the long term consideration, over 70% of comminution cost goes into providing energy; thus, reducing energy cost is expected to reduce the overall concentration cost significantly. This article considers the various relationships between size reduction and energy consumption in most comminution circuits, reviews the various factors that affect energy utilization efficiency in comminution, developed models that integrated the various inefficiency factors in grinding and applied these models to determine the energy required for comminution using a software, ProMax2100, into which some of the energy models have been incorporated. The results show that for the same equipment size, product and feed requirements, a 2-stage crushing and 3-stage grinding circuit gives a difference of 2.347kWh in the energy calculated using Bond’s model only and that obtained with the use of applicable inefficiency factors with Bond’s energy.

Keywords: Factorial models, inefficiency factors, ProMax2100 application, crushing circuits, grinding circuits, wet grinding, dry grinding, actual total energy

INTRODUCTION

Comminution is required for most ore processing operations because the greater number of mineral occurrences is in rock forms in which valuable minerals are locked in fine to coarse grain particles. Even where the ore occurs in loose or friable form, the mineral components may still be coarse enough to require comminution in order to achieve adequate liberation depending on the choice of concentration method to be used. However, this unit operation which consists mainly of crushing and grinding is the most cost intensive unit of any mineral processing plant; and a large portion of the operating cost goes into providing energy for size reduction. In the history of ore processing, very few comminution circuits may have involved only crushing without grinding. Thus with the exception of aggregates production plants, most ore processing plants contain the two comminution units. John and Brain, (2002) and Boyd, (2002) showed that, the cost of providing energy for milling in any plant requiring grinding is usually well over 100% that required for crushing depending on the concentration method used. Thus, it is obvious that achieving reduction in processing cost is significantly dependent on reducing comminution (especially grinding) cost.

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Since most modern comminution circuits are developed from careful research and tests carried out on ore properties, it follows that excluding an aspect of the circuit may not result in cost reduction. The best approach therefore is to determine a method for increasing the efficiency of each unit of the processing plant especially the comminution section.

Many factors contribute to the efficiency or inefficiency of comminution and thus its cost. Among these are the choice of equipment, the choice of circuit, feed and product requirements, plant layout and capacity, condition of operation, maintenance requirements, choice of grinding media, energy tariff and several ore properties including hardness, abrasiveness, percent fine content, and others. All of these factors directly or indirectly affect the overall cost of comminution. Thus, it is important that the inter-relationship between these factors be intimately understood so as to design and select equipment for a comminution circuit that gives the lowest energy cost. This paper therefore looks at the possibility of accurately determining the actual total energy required for comminution based on existing comminution laws by incorporating all known inefficiency factors.

**Crushing Plant Design and Layout**

There are three main steps involved in the design of a good crushing plant. These are circuit design, equipment selection and layout or configuration (Maurice and Kenneth, 2003; Boyd, 2002; DeMull et al., 2002 and Rowland, 2002). Circuit design and equipment selection are dictated by production requirements and other design parameters, but the layout will reflect the preference and experience of the engineering professionals involved especially the consulting engineer (Rowland, 2002). Crushing may be done in four stages but primary and secondary crushing are usually operated in open circuits; though some secondary crushing operations are better done in closed circuits. Tertiary crushing may be done in open or closed circuits but quaternary crushing are traditionally done in closed circuits (Major, 2002; De Mull et al., 2002). Usually primary crushing alone does not produce the required product for downstream processing. Thus secondary crushing is usually required for most operations (Maurice and Kenneth, 2003; Boyd, 2002; Major 2002).

**Grinding Circuits and Layouts**

Although there are wide variety of possible grinding circuits, few of these are significant and have given the best performance for a wide range of ore beneficiation operations (Barratt and Sherman, 2002). Grinding circuit here refers to a combination of primary, secondary, tertiary, reground mills and screens in such a configuration and scale as to produce closely sized feed for the downstream or concentration processes (Major, 2002; Utley, 2002 and Rowland, 2002). Among the several possible mill circuits for variety of geological materials, an autogenous(AG) – ball mill circuit, semi-autogenous (SAG) – ball mill or multistage ball mill circuits are usually preferred.

In modern grinding circuits, rod mills are almost no longer selected to prepare ball mill feed which used to be their primary application. This is partly due to the availability of multi-stage crushing (up to quaternary crushing – though seldom use), followed by ball milling. Also responsible for this trend is the recent increase in the sizes of balls and ball mills (Callow and Meadows; 2002); in addition to greater understanding of autogenous (AG) grinding and increase in the size of autogenous mills with improved power efficiency (Callow and Meadows, 2002; Barratt and Sherman, 2002; De Mull et al., 2002). These options prepare better feed for primary and
secondary ball mills and are more economical (Barratt and Sherman, 2002). Callow and Meadows (2002) shows that the most common and economic circuit in plants around the world is semi-autogenous (SAG) and ball mill layout relationships.

**MATERIALS AND METHODS**

The approach employed in this work is to first review the general and most possible circuits for the common comminution processes. Then a brief review of the major comminution laws and the most critical inefficiency factors that affect milling operations are considered. Thereafter models were developed based on existing laws and these inefficiency factors; and the models for different conditions used directly and indirectly for the determination of actual energy required for some selected comminution circuits. Four comminution circuits (Figures 5 to 8, some of them drawn from existing plants) were used to test the models. The models for the determination of comminution energy without the inefficiency factors were also used to calculate the required comminution energy. The results were then compared. Direct application of the models involved substituting values directly into the model equations while the indirect application involves the use of a computer program Promax2100. This process of application and comparison also helps to validate the models.

**Comminution Laws and Energy Models**

Up till 1959 when Bond’s energy equation for size reduction was developed, Rinttinger and Kick’s theories were effectively used for energy calculation in ore comminution. Although these energy equations are essentially the same, Bond’s equation became the one commonly applied in practical situations. Bond’s equation relates size and energy for 80% oversize and undersize for feed and product respectively. The model developed by Bond is commonly written as in Equation 1 (Will, 2006).

\[
\Sigma = 10w_{ij} \left[ P^{\frac{1}{2}} - F^{\frac{1}{2}} \right] \tag{1}
\]

Where, \( \Sigma \) is the energy in kWh, \( w_{ij} \) is work index in kWh/ton (determined from a grindability test), \( P \) and \( F \) are product and feed sizes in microns. This equation has been the basis for the development of several comminution models used for modern circuit simulation by many software programmes developed for most ball and rod mills as well as for crushing plants. Thus, the total energy \( \Sigma_T \) required for comminution in a plant is represented by Equation 2

\[
\Sigma_T = \Sigma_{CR} + \Sigma_{GR} \tag{2}
\]

Where \( \Sigma_{CR} \) is total energy for all crushing stages (kWh), and \( \Sigma_{GR} \) is total energy for grinding. In Bond’s model format, equation 2 is written as Equation 3. The grinding component of Equation 2 is usually applied to wet grinding since dry grinding require extra energy in which case a dry grinding factor must be applied for correction.

\[
\Sigma_T = 10w_{Ci} \left[ P^{\frac{1}{2}} - F^{\frac{1}{2}} \right]_{CR} + 10w_{Gi} \left[ P^{\frac{1}{2}} - F^{\frac{1}{2}} \right]_{GR} \tag{3}
\]

Here \( w_{Ci} \) and \( w_{Gi} \) are the work indexes for crushing and grinding respectively in kWh/ton. The various inefficiency factors will be incorporated into equation (3) one after another to develop the final energy models for ball operations.
Developing the Models

According to Rowland, (2002); Derek and Mark, (2002), and Callow and Moon, (2002), many other factors apart from those considered by early comminution theories affect the efficiency of the comminution process and thus the energy required. The efficiency of crushing is usually affected by few factors most of which are adequately care for by good understanding of ore properties from compositional analysis, careful selection of crushing equipment, and effective control of feed and products. But grinding is affected by several inefficiency factors and their causative conditions must be properly internalized in the grinding process and energy equations so as to accurately calculate the energy consumed in the milling process. Thus, the crushing energy part of Equation 2 can be adequately represented by Bond’s energy model; but the model for grinding part must have all inefficiencies factors incorporated for both ball mill and rod mill applications in wet or dry conditions. Autogenous and semi-autogenous mills also usually obey comminution law, thus, the inefficiency factors also apply to them.

The eight inefficiency factors to be applied to the calculated grinding energy to allow for variations from the specific conditions and optimum feed sizes were first published in 1973 by Rowland, Jr (Rowland 2002; Nitta et al., 2002) and included dry grinding inefficiency factor (\(F_{DR}\)), open circuit ball milling inefficiency factor (\(F_{CO}\)), mill diameter inefficiency Factor (\(F_{D}\)), feed oversize inefficiency factor (\(F_{SO}\)), inefficiency factor for fine grinding (\(F_{N}\)) in ball mills to product sizes finer than 80% passing 200mesh (75 microns), inefficiency factor for high or low ratio of reduction rod milling (\(F_{rd}\)), low ratio of reduction ball milling inefficiency factor\((F_{rb})\), and rod milling inefficiency factor. Each of these inefficiency factors applies to the specific condition for which it is defined. Since grinding is usually done in closed or open circuits, and either wet or dry condition, this article considers the incorporation of the inefficiency factors under these conditions.

Dry and Wet Grinding in Closed Circuit

Dry Grinding Efficiency Factor \((F_{dg})\): For the same range of work, it has been established (Rowland 2002) that dry grinding requires about 1.3 times as much power as wet grinding. Thus, the inefficiency multiplier for dry grinding for both rod and ball milling is 1.3. Except for few applications, rod milling is usually done wet, thus this inefficiency factor applies most effectively to dry ball milling (Rowland 2002). Thus, applying this factor to Equation 2 gives:

\[\Sigma_T = \Sigma_{CR} + \Sigma_{GR} \]

and,

\[\Sigma_T = \Sigma_{CR} + 1.3\Sigma_{GR} \]

Diameter Efficiency Factor \((F_{D})\): Rowland (2002) used a ball mill of inside shell diameter 2.59 meters or 85 inches diameter and inside liners 2.44 meters as the base mill diameter and computed the diameter inefficiency factor \((F_{D})\) as:

\[F_{D} = \left[ \frac{2.44}{D} \right]^{0.2} \]

Where \(D\) is the inside liners mill diameter

Thus the Diameter Efficiency factors for most of the common mills are computed by Rowland (2002) as shown in Table 1. It can be observed from the table that as the mill diameter increases, the value of the diameter efficiency multiplier reduces gradually to 1.00 for the base diameter of 2.44 meters (inside liners) and 2.59 meters inside shell, showing that efficiency also gradually increase with increasing diameter. Below the base diameter, the value of the factor
Table 1: Mill diameter Efficiency Factor

<table>
<thead>
<tr>
<th>S/N</th>
<th>Mill Diameter inside shell (m)</th>
<th>Mill Diameter inside liners (m)</th>
<th>Diameter Efficiency factor multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.914</td>
<td>0.79</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>0.88</td>
<td>1.23</td>
</tr>
<tr>
<td>3</td>
<td>1.22</td>
<td>1.10</td>
<td>1.17</td>
</tr>
<tr>
<td>4</td>
<td>1.52</td>
<td>1.40</td>
<td>1.12</td>
</tr>
<tr>
<td>5</td>
<td>1.83</td>
<td>1.71</td>
<td>1.075</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
<td>1.82</td>
<td>1.06</td>
</tr>
<tr>
<td>7</td>
<td>2.13</td>
<td>1.98</td>
<td>1.042</td>
</tr>
<tr>
<td>8</td>
<td>2.44</td>
<td>2.29</td>
<td>1.014</td>
</tr>
<tr>
<td>9</td>
<td>2.59</td>
<td>2.44</td>
<td>1.00 - Base</td>
</tr>
<tr>
<td>10</td>
<td>2.74</td>
<td>2.59</td>
<td>0.992</td>
</tr>
<tr>
<td>11</td>
<td>2.90</td>
<td>2.74</td>
<td>0.972</td>
</tr>
<tr>
<td>12</td>
<td>3.0</td>
<td>2.85</td>
<td>0.970</td>
</tr>
<tr>
<td>13</td>
<td>3.05</td>
<td>2.90</td>
<td>0.966</td>
</tr>
<tr>
<td>14</td>
<td>3.20</td>
<td>3.05</td>
<td>0.956</td>
</tr>
<tr>
<td>15</td>
<td>3.35</td>
<td>3.20</td>
<td>0.948</td>
</tr>
<tr>
<td>16</td>
<td>3.51</td>
<td>3.35</td>
<td>0.939</td>
</tr>
<tr>
<td>17</td>
<td>3.66</td>
<td>3.51</td>
<td>0.931</td>
</tr>
<tr>
<td>18</td>
<td>3.81</td>
<td>3.66</td>
<td>0.923</td>
</tr>
<tr>
<td>19</td>
<td>3.96</td>
<td>3.81</td>
<td>0.914</td>
</tr>
<tr>
<td>20</td>
<td>4.00</td>
<td>3.85</td>
<td>0.914</td>
</tr>
</tbody>
</table>

falls below 1.00 and remain constant at 0.9414 for the largest diameter of 4.0 meters and 3.81 meters inside shell and inside liners respectively. Thus, this factor is usually neglected when selecting a mill in which the factor is less than one (i.e. < 1.0), (Rowland, 2002, Callow and Moon 2002).

Determining the total energy for wet grinding with diameter inefficiency multiplier in closed circuit requires that Equation 6 be applied as multiplier to the grinding component of Equation 4 which gives Equation 7.

\[ \Sigma_T = \Sigma_{CR} + \left[ \frac{2.44}{D} \right]^{0.2} \Sigma_{GR} \]

\[ \Sigma_T = \Sigma_{CR} + 1.2 \left[ \frac{1}{D^{0.2}} \right] \Sigma_{GR} \]

For dry grinding in closed circuit, Equation 8 introduces the dry grinding inefficiency factor (1.3) as multiplier to Equation 7.

\[ \Sigma_T = \Sigma_{CR} + (1.3)(1.2) \left[ \frac{1}{D^{0.2}} \right] \Sigma_{GR} \]

\[ \Sigma_T = \Sigma_{CR} + 1.56 \left[ \frac{1}{D^{0.2}} \right] \Sigma_{GR} \]

\[ \Sigma_T = \Sigma_{CR} + \left( \frac{W_i}{7} \right) \left( \frac{F - F_0}{F_0} \right) \]

\[ \Sigma_T = \Sigma_{CR} + \left( \frac{W_i}{7} \right) \left( \frac{F - F_0}{F_0} \right) \]

The Oversized Feed Efficiency Factor (\( F_{SO} \)): Rod and ball milling are efficient when fed with the optimum feed size. However, when being fed with sizes coarser than the optimum feed size which is usually the situation with most run-of-mines, an oversize feed factor (\( F_{SO} \)) should be introduced as multiplier for energy correction. This is most frequently used with a single stage ball milling (Rowland, 2002 and Callow and Moon, 2002). The oversize feed inefficiency factor is the only factor that is directly related to work index as shown by Equation 9.

\[ F_{SO} = \frac{R_r}{R_r} \left( \frac{F - F_0}{F_0} \right) \]

Where \( W_i \) = Work index (as determined by Bond equation using grindability test at the desired grind)

\[ R_i = \text{Reduction Ratio} = \frac{F}{P} \]

\( F_o = \text{Optimum Feed size obtained from Equations 10 and 11 for rod mill and ball mill respectively.} \)

For Rod milling:

\[ F_o = 16,000 \left( \frac{13}{W_i} \right)^{0.5} \]  

For Ball Milling:

\[ F_o = 4,000 \left( \frac{13}{W_i} \right)^{0.5} \]  

For work index \( W_i \) in each of Equations 10 and 11 above, Rowland (2002) opined that:

i. work index from Bond impact test or a rod mill grindability test (whichever is higher) should be used in Equation 10 and

ii. ball milling work index from a rod mill grindability test should be used in Equation 11 since this represents the coarse fraction of the feed; but if not available, ball mill grindability test results should be used.

Since it is sometimes difficult to maintain a closely sized feed for milling operations, the optimum feed size should always be determined and used to regulate mill discharge or apply this inefficiency factor to the calculated Bond energy.

Thus applying this factor to wet grinding in closed circuit with other applicable factors, Equation 12 incorporates this factor into Equation 7:

\[ \Sigma_T = \Sigma_{CR} + 1.2 \left( \frac{1}{D^{0.2}} \right) \]

\[ R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right) \frac{\Sigma_{GR}}{R_r} \]

For dry grinding in closed circuit, 13 applies.

\[ \Sigma_T = \Sigma_{CR} + 1.56 \left( \frac{1}{D^{0.2}} \right) \]

\[ R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right) \frac{\Sigma_{GR}}{R_r} \]

Fineness of grind factor (\( F_r \)): This factor applies when, grinding to product size finer than 80% passing 75um (i.e 20mesh). This factor according to Rowland, (2002) and Callow and Moon, (2002) is determined using Equation 14.

\[ F_N = \frac{P + 10.3}{1.145P} \]

Where \( P = \text{is desired product size} \)

Equations 15 and 16 apply this inefficiency factor to Equations 12 and 13 for wet and dry grinding in closed circuit respectively.

\[ \Sigma_T = \Sigma_{CR} + 1.2 \left( \frac{P + 10.3}{1.145P} \right) \left[ \frac{1}{D^{0.2}} \right] \]

\[ R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right) \frac{\Sigma_{GR}}{R_r} \]

For dry grinding in closed circuit, 16 apply.

\[ \Sigma_T = \Sigma_{CR} + 1.56 \left( \frac{P + 10.3}{1.145P} \right) \left[ \frac{1}{D^{0.2}} \right] \]

\[ R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right) \frac{\Sigma_{GR}}{R_r} \]
High or Low Reduction Ratio Rod Milling ($F_{rr}$): When reduction ratio for rod milling is low or high then grinding is inefficient as more energy will be required to produce the required product size or there is overgrinding. Then it will be necessary to apply this factor for correction, (Rowland, 2002). As can be imagined from the above, the application of this factor to high reduction ratio will not be necessary as correction can be made by reducing the resident time of materials in the mill thus reducing a particular charge. Hence, this factor always apply to low reduction ratio rod milling and is generally applied for selection of mill when work index (Wi) from rod and ball mill grindability test exceed 7.0. Rowland, (2002) gives the equation to be used as:

$$F_{rr} = \frac{1 + (R_r - R_{ro})^2}{150} \quad \text{.........17}$$

Unless $R_r$ is between $R_{ro} = -2$ or $+2$

Where, $R_r =$ reduction ratio

$$R_{ro} = 8 + \frac{5L}{D} \quad \text{.........18}$$

Where, $L =$ Rod length

$D =$ Diameter inside liners

Low Ratio of Reduction Ball Milling ($F_{rb}$): This factor is often applied when the reduction ratio for ball milling is less than 6. Such situation occurs particularly in regrinding (feed, concentrate and tailings). The equation for determining this according to Rowland, (2002) and Callow and Moon, (2002) is:

$$F_{rb} = \frac{2(R_r - 1.35) + 0.26}{2(R_r - 1.35)} \quad \text{.........19}$$

For wet and dry grinding in closed circuit ball mill with low reduction ratio, equation 19 is introduced as efficiency multiplier to Equations 15 and 16 to develop Equations 20and 21.

$$\Sigma_T = \Sigma_{CR} + 1.2 \left[ \frac{P + 10.3}{1.145P} \right]$$

$$\left[ \frac{1}{D^{0.2}} \right] \frac{2(R_r - 1.35) + 0.26}{2(R_r - 1.35)}$$

$$R_r + (W_r - 7) \left( \frac{F - F_0}{F_0} \right) \frac{1}{R_r}$$

$$\Sigma_{GR} \quad \text{.........20}$$

For dry grinding in closed circuit, Equation 21 applies.

$$\Sigma_T = \Sigma_{CR} + 1.56 \left[ \frac{P + 10.3}{1.145P} \right]$$

$$\left[ \frac{1}{D^{0.2}} \right] \frac{2(R_r - 1.35) + 0.26}{2(R_r - 1.35)}$$

$$R_r + (W_r - 7) \left( \frac{F - F_0}{F_0} \right) \frac{1}{R_r}$$

$$\Sigma_{GR} \quad \text{.........21}$$

Rod Milling Inefficiency Factor: Rowland (2002) and Callow and Moon (2002) show that rod mill performance is affected by feed preparation and feeding of a uniform top size, mill care or maintenance and rod charge. Thus, these factors usually make rod mill operation inefficient and an efficiency factor must therefore be introduced. Rowland (2002) however, showed that this efficiency factor has not been definitely determined numerically or empirically, but in selecting rod mills based on power calculated from grindability test, the following procedure are recommended.

1. When calculating rod mill power for a rod mill - only application, an inefficiency factor of 1.4 should be
used when the feed is to be prepared with open circuit crushing, and 1.2 when the feed is to be prepared with closed circuit crushing. The mill diameter, low or high ratio of reduction, and oversize inefficiency factors must also be applied to the calculated grinding power.

\[ (2) \]

When calculating rod mill power for a rod mill-ball mill circuit and the rod mill feed is produced with open circuit crushing, an inefficiency factor of 1.2 must be applied to the rod mill stage only; but if the rod mill feed will consistently be the same such as produced with closed circuit crushing, then a rod mill inefficiency factor may not be applied but the mill diameter, low or high reduction ratio, and oversize feed factors should be applied to the calculated grinding power.

The criteria for selecting any of these options are determined by ore variability. Thus tests must be conducted on mill feed samples that have been selected according to the mine production plan. The number of grinding lines in a circuit is influenced by plant capacity, and mill size, classification systems and mill discharge rate (Callows and Moon 2002).

**Dry and Wet Grinding in Open Circuit**

**Open Circuit Ball Milling Efficiency Factor** \( (F_{CO}) \): Grinding in open circuit ball mills require extra power as compared to grinding in closed circuit. This may be due to the fact that new feed is constantly being added while product discharged without screening by the circuit. Thus, the amount of extra power required is a function of the degree of control required on the product produced (Rowland, 2002). The inefficiency factors for open circuit grinding are given in Table 2. The basic calculation used in computing values in Table 2 is the work required per Bond Equation for the desired 80% passing size after which the inefficiency factor is applied. Thus for **wet and dry grinding in open circuits**, the open circuit inefficiency factor \((K)\) is applied to Equations 4, 5, 7, 8, 12, 13, 15, 16, 20 & 21 to generate a another set of models shown in Equations 22 to 31.

**Table 2: Open Circuit Grinding Efficiency**

<table>
<thead>
<tr>
<th>Multiplier or Factor (( F_{CO} = K ))</th>
<th>Product size control % passing</th>
<th>Inefficiency (Factor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.035</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>92</td>
<td>1.46</td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>1.70</td>
<td></td>
</tr>
</tbody>
</table>

Thus for **wet and dry grinding in open circuit**, the required energy is:

\[
\Sigma_T = \Sigma_{CR} + K \Sigma_{GR} \quad \text{.............22}
\]

and,

\[
\Sigma_T = \Sigma_{CR} + 1.3K \Sigma_{GR} \quad \text{.............23}
\]

Energy requirement for **wet grinding with diameter inefficiency factor in open circuit** requires Equation 24.

\[
\Sigma_T = \Sigma_{CR} + 1.2 \left[ \frac{K}{D^{0.2}} \right] \Sigma_{GR} \quad \text{.............24}
\]

For **dry grinding in open circuit** with diameter inefficiency factor, Equation 25 is applicable

\[
\Sigma_T = \Sigma_{CR} + 1.56 \left[ \frac{K}{D^{0.2}} \right] \Sigma_{GR} \quad \text{.............25}
\]

For **wet grinding in open circuit** with other applicable factors, Equation 26 is used to determine the total actual energy required with oversize feed inefficiency factor.
\[
\Sigma_T = \Sigma_{CR} + 1.2 \left[ \frac{K}{D^{0.2}} \right]
\]
\[
\frac{R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right)}{R_r} \Sigma_{GR} \ldots \ldots \ldots 26
\]

For dry grinding in open circuit with oversize feed inefficiency factor, Equation 27 applies.

\[
\Sigma_T = \Sigma_{CR} + 1.56 \left[ \frac{K}{D^{0.2}} \right]
\]
\[
\frac{R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right)}{R_r} \Sigma_{GR} \ldots \ldots \ldots 27
\]

Equations 28 and 29 apply to fineness of grind inefficiency factor to determine the total actual energy for wet and dry grinding in open circuit respectively.

\[
\Sigma_T = \Sigma_{CR} + 1.2 \left[ \frac{P + 10.3}{1.145P} \right] \left[ \frac{K}{D^{0.2}} \right]
\]
\[
\frac{R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right)}{R_r} \Sigma_{GR} \ldots \ldots \ldots 28
\]

\[
\Sigma_T = \Sigma_{CR} + 1.56 \left[ \frac{P + 10.3}{1.145P} \right] \left[ \frac{K}{D^{0.2}} \right]
\]
\[
\frac{R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right)}{R_r} \Sigma_{GR} \ldots \ldots \ldots 29
\]

For wet and dry grinding in open circuit ball mill with low reduction ratio factor, Equations 30 and 31 are used to calculate the actual energy with all other inefficiency factors.

\[
\Sigma_T = \Sigma_{CR} + 1.2 \left[ \frac{P + 10.3}{1.145P} \right] \left[ \frac{K}{D^{0.2}} \right]
\]
\[
\frac{2(\bar{R}_r - 1.35) + 0.26}{2(\bar{R}_r - 1.35)} \left[ \frac{R_r + (W_i - 7) \left( \frac{F - F_0}{F_0} \right)}{R_r} \right] \Sigma_{GR} \ldots \ldots \ldots 31
\]

These models are stepwise applications of the inefficiency factors with an overall format of the form shown in Equation 32. Thus, where a particular factor does not apply, ProMax multiplies the calculated Bond’s energy by 1 to eliminate the effect of that factor.

\[
\Sigma_T = 10w_G \left[ P_c^{\frac{1}{2}} - F_c^{\frac{1}{2}} \right]
\]
\[
+ 10w_G \left( F_{D \phi} F_{CO} F_{D} F_{SO} F_{N} F_{R R} \right) \left[ P_G^{\frac{1}{2}} - F_G^{-\frac{1}{2}} \right] \ldots \ldots 32
\]

**ProMax2100 Simulator**

ProMax2100 incorporates these models for calculating the total actual energy required for size reduction in any comminution circuit. ProMax 2100 is mineral processing plant design software developed by the mineral processing unit of the Mining Engineering Department, the Federal University of Technology, Akure, Nigeria. ProMax is designed for simulating mineral processing plant from start to finish. Though not yet fully developed, it automatically selects and proportions equipment for every stage of a processing plant based on some sets of built-in and input data from results plants bench scale tests, plant capacity and products requirements.
As shown in the ProMax user interface in Figures 1-4, the programme is design to be able to break each models into the two major unit operation of comminution operation, namely crushing and grinding, and calculate the energy separately or together. Details of the development of ProMax are provided in another article. A demo of the software will soon be available on the internet for on-line simulation of parts of a mineral processing plant.

Figure 1: ProMax 2100 User’s Interface for Crusher Selection

Figure 2: ProMax 2100 User’s Interface for Crushing Stage Definition
Figure 3: ProMax 2100 User’s Interface for Selection Mill and Grinding Stages

Figure 4: ProMax 2100 User’s Interface for Energy Calculation

RESULTS AND DISCUSSION

ProMax was used to apply the inefficiency factors to the comminution circuits shown in Figures 5 to 8 to calculate the total actual energy required in each application. Some of these circuits are drawn from existing plants. The energy required without the application of the factors was first determined by applying Bond’s model directly. The results were then compared...
Table 3: Communion Energy Generated by ProMax for the Circuit in Figure 5

<table>
<thead>
<tr>
<th>ProMax 21</th>
<th>Energy Required for Communion Units (Option A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Stage</td>
</tr>
<tr>
<td>Crushing</td>
<td>Pcy</td>
</tr>
<tr>
<td></td>
<td>Sec.</td>
</tr>
<tr>
<td>Grinding</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>PRC-SD</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Communion Energy Generated by ProMax for the Circuit in Figure 6

<table>
<thead>
<tr>
<th>ProMax 21</th>
<th>Energy Required for Communion Units (Option A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Stage</td>
</tr>
<tr>
<td>Crushing</td>
<td>Pcy</td>
</tr>
<tr>
<td></td>
<td>Sec.</td>
</tr>
<tr>
<td></td>
<td>Ter.</td>
</tr>
<tr>
<td>Grinding</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
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<td></td>
</tr>
<tr>
<td>PRC-SD</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Communion Energy Generated by ProMax for the Circuit in Figure 7

<table>
<thead>
<tr>
<th>ProMax 21</th>
<th>Energy Required for Communion Units (Option A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Process</td>
<td>Stage</td>
</tr>
<tr>
<td>Crushing</td>
<td>Pcy</td>
</tr>
<tr>
<td></td>
<td>Sec.</td>
</tr>
<tr>
<td></td>
<td>Ter.</td>
</tr>
<tr>
<td>Grinding</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>PRC-SD</td>
<td></td>
</tr>
</tbody>
</table>
Table 6: Comminution Energy Generated by ProMax for the Circuit in Figure 8

<table>
<thead>
<tr>
<th>Process</th>
<th>Stage</th>
<th>Type</th>
<th>Avg. Feed Size (mm)</th>
<th>Avg. Product Size (mm)</th>
<th>Bond’s Energy (kWh/t)</th>
<th>Applicable Inefficiency Multiplier</th>
<th>Actual Energy (kWh/t)</th>
<th>Energy Tariff ($/kWh)</th>
<th>Energy Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crushing</td>
<td>Pry</td>
<td>Gyratory</td>
<td>1800</td>
<td>350</td>
<td>92</td>
<td>NA</td>
<td>92</td>
<td>0.25</td>
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</tr>
<tr>
<td></td>
<td>Sec.</td>
<td>Jaw</td>
<td>350</td>
<td>200</td>
<td>53</td>
<td>NA</td>
<td>53</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Grinding</td>
<td>1</td>
<td>CC SAG</td>
<td>200</td>
<td>75</td>
<td>170</td>
<td>NA</td>
<td>205</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>OC Ball</td>
<td>75</td>
<td>25</td>
<td>321</td>
<td>NA</td>
<td>386</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>CC Ball</td>
<td>25</td>
<td>45μ</td>
<td>17,125</td>
<td>NA</td>
<td>19,202</td>
<td></td>
<td></td>
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<tr>
<td>PRC-SD</td>
<td></td>
<td></td>
<td>17591</td>
<td></td>
<td>19938</td>
<td>025</td>
<td>4983.500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5: A two Stage Crushing and Single Stage Grinding Circuit

Figure 6: A three-Stage Crushing and Two-Stage Grinding Circuit
with those obtained with the application of the inefficiency factors. The work index used for crushing \(W_c\) is 9.78kWh/t and that for grinding \(W_g\) is 12.0kWh/t (Maurice and Kenneth, 2003). Tables 3-6 are obtained from ProMax results and edited for this article.

It is assumed that grinding in all four circuits is done wet in a mill of suitable diameter to products sizes passing 80% require grind size. Thus, dry grinding inefficiency factor \(F_{ip}\) and diameter inefficiency factor \(F_P\) are not applicable to the circuits. Fineness of grind factor \(F_{G}\) applies only in the third grinding stage of the circuit in Figure 8 where grinding is done to 80% passing 45microns. Oversize feed factor \(F_{so}\) applies to the grinding in Figure 5, and the final grinding stages of Figures 6 and 7. Open circuit factor \(F_{OC}\) and low ratio of reduction \(F_{PR}\) applies to the first grinding stages of Figures 6-8 where reduction ratios are as low as 2.5, 4 and 3 respectively. Since all products are ground to 80% passing required size, the open circuit efficiency factor is 1.2 as shown in Table 2. The applicable inefficiency factor for each circuit is shown in the corresponding grinding stages in Tables 3-6.

The results shown in Tables 3-6 highlight the importance of given serious attention to grinding circuits during the design of a mineral processing plant, especially plants that require fine grinding to achieve adequate liberation for effective recovery. For some obvious reasons the energy calculated with Bond’s model does not represent the actual energy used in comminution. Although Bond’s model itself is not deficient, but it does not take into account, the several conditions that may affect the grinding process which in turn result in under-estimation or over-estimation of the energy. Bond’s model development was based on particle sorting resulting from changes in the internal geometry of the material. The model did not consider the effects of such factors as the grinding condition, mill layout and circuit design, mill type, size and mechanism, mode of
energy application, effect of feed size and others because these factors although have some effects on the efficiency of the comminution process, are external to the properties of the material on which Bond’s model was based. The harmonization of these factors obviously determine the total actual energy used in comminution (Wills, 2006 and Rowland, 2002). Calculating energy without the application of these factors give lower value of energy than is actually used in the size reduction process. The results presented in Tables 3-6 show that the value of the unaccounted energy as a result of not applying inefficiency factors is significant. For example the energy difference between the use of Bond’s model only and the application of the inefficiency factors to the model for the single stage grinding in Figure 5 is 1,368kWh; and for the two stage crushing with a three-stage grinding in Figure 8 is 2,347kWh. The seriousness of exclusion of the inefficiency factors in the grinding process becomes obvious when you compare these differences with the energy expended on crushing as a whole. In Figures 5 and 8, the total energy used in all stages of crushing for the two circuits are 355kWh and 145kWh respectively. The unaccounted energy (i.e energy differences due to the application of inefficiency factors – 1,368kWh : 355kWh and 2,347kWh : 145kWh) are 385% and 1600% respectively of the total energy expended without application of the inefficiency factors. Surely this is not a situation to be neglected as far as engineering design is concerned.

It is also clearly show here that the energy required for crushing is seriously insignificant when compare to that required for grinding thus emphasizing the need for more attention in the designing of the grinding unit. This analysis assumed that all other factors that are not applied here meet the requirements for their exclusion otherwise the actual energy would have been higher.

CONCLUSION

Since the success of any mineral processing operation depends on the effectiveness of the comminution process, it is important that adequate attention is given to the design and layout of the comminution circuit especially the grinding unit. Equipment should be well selected to eliminate the effects of the inefficiency conditions due to the use of inappropriate equipment. For instance, selecting a mill of adequate size and geometry will eliminate the effects of diameter inefficiency factor. Grinding conditions should be clearly defined and the appropriate inefficiency factors applied.

REFERENCES


Doug Halbe and Derek Barratt, Society for Mining, Metallurgy and Exploration Inc. (SME), Littleton, USA.


