Fjrs.futa.edu.ng

FUTA Journal of Research in Sciences

ISSN: 2315 – 8239 (Print); E-ISSN: 2489 - 0413

FUTA Journal of Research in Sciences, Vol. 13 (2), October, 2017:286-294

INFLUENCE OF PRESTRESS ON THE VIBRATION OF CANTILEVER PLATE RESTING ON BI-PARAMETRIC FOUNDATION

Awodola, T. O.

Department of Mathematical Sciences
Federal University of Technology, Akure, Nigeria
Email address: oluthomas71@yahoo.com

ABSTRACT

The influence of prestress on the response to moving concentrated masses of prestressed cantilever rectangular plate resting on bi-parametric (Pasternak) elastic foundation is investigated in this work. A technique based on separation of variables is used to reduce the governing partial differential equation with variable and singular coefficients to a set of coupled second order ordinary differential equations which are decoupled and simplified using a modification of the Struble's technique, the simplified equations are solved using the method of integral transformations. The numerical results in plotted curves show that the response amplitudes of the cantilever plate decrease as the value of the axial force (prestress) in x-direction (N_x) increases; the axial force in y-direction (N_y) also produces the same effect for both cases of moving force and moving mass problems of the prestressed cantilever rectangular plate resting on Pasternak elastic foundation. However, higher values of N_y are required for a noticeable effect than N_x, this implies that N_x which is the prestress in the x-direction (the assumed direction of motion) has greater influence on the deflection of the cantilever plate than N_y. Also, for fixed values of N_x and N_y, the transverse deflections of the rectangular plates under the actions of moving masses are higher than those when only the force effects of the moving loads are considered, Analysis of the closed form solution shows that the critical speed for the moving mass problem is reached prior to that of the moving force problem which implies that the moving force solution is not a safe approximation to the moving mass problem of the prestressed cantilever rectangular plate resting on Pasternak elastic foundation.

Key words: Cantilever, Prestress, Pasternak, Critical Speed, Moving Force, Moving Mass.

INTRODUCTION

The problem of structures constantly acted upon by moving masses is very common in literature (Fryba 1972, Oni 1991, Inglis 1934, Gbadeyan and Aiyesimi 1990, Sadiku and Leipholz 1981, Gbadeyan and Oni 1995, Jaeger and Strarfield 1979). In most analytical studies in Engineering and Mathematical Physics, structural members are commonly modeled as a beam or as a plate. Aside the problem arising from the inclusion of the inertia terms in moving mass problems, difficulties often arise from the type of specified end-conditions. There are four classical boundary conditions that are commonly of practical interest to an applied Mathematician or an Engineer; these are simply supported end conditions, fixed / clamped end conditions, free end conditions and sliding end conditions (Fryba 1972, Jaeger and Strarfield 1979). When the structure is clamped at one end and free at the other end we have the clamped-free (cantilever) condition.

Researchers have made several efforts in the study of dynamics of structures under moving loads (Shadnam et al 2001, Oni 2004, Oni and Omolofe 2005, Oni and Awodola 2003, Omer and Aitung 2006, Adams 1995, Savin 2001, Jia-Jang 2006, Awodola 2014). Often, engineers create artificial stresses in structures before loading, such artificial stresses are forces which may act axially or otherwise. When they act axially, they are called axial forces. The axial force interacts with the lateral displacement to produce an additional term (Clough and Penziens 1975). This additional term due to the axial force (prestress) increases the complexity of the problem.

The foundation model based on Winkler's approximation model is very common in literature, whereas a more accurate bi-parametric (Pasternak) subgrade model which in addition to subgrade modulus incorporates the shear effect of the foundation should be used rather than the Winkler's approximation model. Eisenberger and Clastornik (1987) presented two methods for the solution of beams on two-parameter elastic foundation. Also, Gbadeyan and Oni (1992) studied the dynamic analysis of an elastic plate continuously supported by an elastic Pasternak foundation traversed by an arbitrary number of concentrated masses.

In most of the investigations in literature on vibration of rectangular plate under moving loads and resting on elastic foundations, work has been restricted to cases when the plate is not prestressed. The more complicated case, when the plate is prestressed has been neglected, where this is considered, work has been restricted to the simplest forms of the problem when the structure is simply supported (Awodola 2014) or when the foundation model is based on the simple and common Winkler's approximation model, other classical boundary conditions such as clamped-free (cantilever) ends condition has not been considered. This paper is therefore concerned with the problem of assessing the influence of prestress on the vibration of prestressed cantilever (clamped-free) rectangular plate resting on bi-parametric (Pasternak) elastic foundation.

GOVERNING EQUATION

The equation governing the dynamic transverse displacement Z(x,y,t) of a prestressed rectangular plate when it is resting on a bi-parametric (Pasternak) elastic foundation and traversed by several concentrated masses M_i moving with velocity c_i (issuing from point y = s on the y – axis) is the partial differential equation given by

$$D\nabla^4 Z(x,y,t)+\mu\frac{\partial^2 Z(x,y,t)}{\partial t^2}=\mu R_0\left[\frac{\partial^4}{\partial t^2\partial x^2}+\frac{\partial^4}{\partial t^2\partial y^2}\right]Z(x,y,t)$$
$$+\left[N_x\frac{\partial^2 Z(x,y,t)}{\partial x^2}+N_y\frac{\partial^2 Z(x,y,t)}{\partial y^2}\right]-F_0Z(x,y,t)$$
$$+G_0\left[\frac{\partial^2 Z(x,y,t)}{\partial x^2}+\frac{\partial^2 Z(x,y,t)}{\partial y^2}\right]+\sum_{i=1}^{N}\Big[M_i g\delta(x-c_it)\delta(y-s)$$
$$-M_i\left(\frac{\partial^2}{\partial t^2}+2c_i\frac{\partial^2}{\partial t\partial x}+c_i^2\frac{\partial^2}{\partial x^2}\right)Z(x,y,t)\delta(x-c_it)\delta(y-s)\Big] \quad (1)$$

where $D=\dfrac{Eh^2}{12(1-v)}$ is the bending rigidity of the plate, ∇^2 is the two-dimensional Laplacian operator, h is the plate's thickness, E is the Young's Modulus, v is the Poisson's ratio $(v<1)$, μ is the mass per unit area of the plate, N_x and N_y are the axial forces in x and y directions respectively, R_0 is the Rotatory inertia correction factor, F_0 is the foundation's stiffness, G_0 is the shear modulus, x and y are respectively the spatial coordinates in x and y directions and t is the time coordinate.

For a cantilever rectangular plate clamped at the edge x = 0, x = L_x and free at the edge y = 0, y = L_y, the boundary conditions are given by

$$Z(0,y,t)=0, \qquad Z(L_X,y,t)=0 \quad (2)$$

$$\frac{\partial^2 Z(x,0,t)}{\partial y^2}=0, \quad \frac{\partial^2 Z(x,L_Y,t)}{\partial y^2}=0 \quad (3)$$

$$\frac{\partial Z(0,y,t)}{\partial x}=0, \quad \frac{\partial Z(L_x,y,t)}{\partial x}=0 \quad (4)$$

$$\frac{\partial^3 Z(x,0,t)}{\partial y^3}=0, \quad \frac{\partial^3 U(x,L_Y,t)}{\partial y^3}=0 \quad (5)$$

Thus for the normal modes

$$\Psi_{ni}(0)=0, \qquad \Psi_{ni}(L_X)=0 \qquad (6)$$

$$\frac{\partial^2\Psi_{nj}(0)}{\partial y^2}=0, \quad \frac{\partial^2\Psi_{nj}(L_Y)}{\partial y^2}=0 \qquad (7)$$

$$\frac{\partial\Psi_{ni}(0)}{\partial x}=0, \quad \frac{\partial\Psi_{ni}(L_x)}{\partial x}=0 \qquad (8)$$

$$\frac{\partial^3\Psi_{nj}(0)}{\partial y^3}=0, \quad \frac{\partial^3\Psi_{nj}(L_Y)}{\partial y^3}=0 \qquad (9)$$

The initial conditions, without any loss of generality, is taken as

$$Z(x,y,0)=0=\frac{\partial Z(x,y,0)}{\partial t} \qquad (10)$$

ANALYTICAL APPROXIMATE SOLUTION

Evidently, an exact closed form solution of the above fourth order partial differential equation (1) does not exist. Consequently, an approximate solution is sought. Thus, the technique based on separation of variable described in (Awodola 2015, Awodola and Oni 2013) is employed.

This versatile technique requires that the solution of equation (1) takes the form

$$Z(x,y,t)=\sum_{n=1}^{\infty}\phi_n(x,y)T_n(t) \qquad (11)$$

where ϕ_n are the known eigen functions of the plate with the same boundary conditions and have the form of (Shadnam et al 2001)

$$\nabla^4\phi_n-\omega_n^4\phi_n=0 \qquad (12)$$

where

$$\omega_n^4=\frac{\Omega_n^2\mu}{D} \qquad (13)$$

Ω_n, n = 1, 2, 3, … , are the natural frequencies of the dynamical system and $T_n(t)$ are amplitude functions which have to be calculated.

The term of G_0 present in equation (1) makes the equation more complex and cumbersome than we have in (Awodola 2015). Nevertheless, following the same procedures as in (Awodola 2015, Awodola and Oni 2013), equation (1) is transformed to become

$$\frac{d^2T_n(t)}{dt^2}+\alpha_n^2T_n(t)-\frac{1}{P^*}\sum_{q=1}^{\infty}\Bigg\{R_0P_1^*\frac{d^2T_q(t)}{dt^2}$$
$$-\frac{1}{\mu}\left(F_0P_2^*-N_xk^0-N_yk^1-G_0P_1^*\right)T_q(t)$$
$$-\sum_{i=1}^{N}\frac{M_i}{L_XL_Y\mu}\Bigg[2\Bigg(\frac{P_3^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi c_it}{L_X}P_3^{***}(j)$$
$$+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi c_it}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\Bigg)\frac{d^2T_q(t)}{dt^2}$$
$$+4c_i\Bigg(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi c_it}{L_X}P_4^{***}(j)$$
$$+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi c_it}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\Bigg)\frac{dT_q(t)}{dt}$$
$$+2c_i^2\Bigg(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi c_it}{L_X}P_5^{***}(j)$$
$$+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi c_it}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\Bigg)T_q(t)\Bigg]\Bigg\}$$
$$=\sum_{i=1}^{N}\frac{M_ig}{P^*\mu}\phi_p(c_it,s) \qquad (14)$$

where $\alpha_n^2=\dfrac{D\omega_n^4}{\mu}$,

$$k^0=\int_0^{L_X}\int_0^{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$k^1=\int_0^{L_X}\int_0^{L_Y}\phi_{n,yy}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_1^*=k^0+k^1,$$

$$P_2^*=\int_0^{L_X}\int_0^{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^*=\int_0^{L_X}\int_0^{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_3^{****}(j,k)=$$
$$\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_n(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{*}=\int_0^{L_X}\int_0^{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_4^{****}(j,k)=$$
$$\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,x}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^{*}=\int_0^{L_X}\int_0^{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^{**}(k)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

$$P_5^{***}(j)=\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx$$

$$and\ \ P_5^{****}(j,k)=$$
$$\int_0^{L_X}\int_0^{L_Y}\cos\frac{j\pi x}{L_X}\cos\frac{k\pi y}{L_Y}\phi_{n,xx}(x,y)\phi_p(x,y)\,dy\,dx,$$

Equation (14) is the transformed equation governing the problem of the rectangular plate on a Pasternak elastic foundation.

In what follows, $\phi_n(x,y)$ are assumed to be the products of the beam functions $\psi_{ni}(x)$ and $\psi_{nj}(y)$ (Lee and Ng 1996). That is

$$\phi_n(x,y)=\psi_{ni}(x)\psi_{nj}(y) \qquad (15)$$

These beam functions can be defined respectively, as

$$\psi_{ni}(x)=\sin\frac{\Omega_{ni}x}{L_X}+A_{ni}\cos\frac{\Omega_{ni}x}{L_X}$$
$$+B_{ni}\sinh\frac{\Omega_{ni}x}{L_X}+C_{ni}\cosh\frac{\Omega_{ni}x}{L_X} \qquad (16)$$

and

$$\psi_{nj}(y)=\sin\frac{\Omega_{nj}y}{L_Y}+A_{nj}\cos\frac{\Omega_{nj}y}{L_Y}$$
$$+B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y}+C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y} \qquad (17)$$

where A_{ni}, A_{nj}, B_{ni}, B_{nj}, C_{ni} and C_{nj} are constants determined by the boundary conditions. Ω_{ni} and Ω_{nj} are called the mode frequencies.

Thus for the single mass M with velocity c equation (14) reduces to

$$\frac{d^2T_n(t)}{dt^2}+\alpha_n^2T_n(t)-\frac{1}{P^*}\sum_{q=1}^{\infty}\Bigg\{R_0P_1^*\frac{d^2T_q(t)}{dt^2}$$
$$-\frac{1}{\mu}\left(F_0P_2^*-N_xk^0-N_yk^1-G_0P_1^*\right)T_q(t)$$
$$-\Gamma\Bigg[2\left(\frac{P_3^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)\right.$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\frac{d^2T_q(t)}{dt^2}$$
$$+4c\left(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j)\right.$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dT_q(t)}{dt}$$
$$+2c^2\left(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)\right.$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)T_q(t)\Bigg]\Bigg\}$$
$$=\frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (18)$$

where $\Gamma=\dfrac{M}{L_XL_Y\mu}$

Equation (18) is the fundamental equation of our problem when the prestressed rectangular plate has arbitrary end support conditions. In what follows, we shall discuss two special cases of the equation (18) namely; the moving force and the moving mass problems.

CASE I:
MOVING FORCE PROBLEM

By setting Γ = 0 in equation (18), an approximate model of the differential equation describing the response of a prestressed rectangular plate resting on a Pasternak elastic foundation and traversed by a moving force would be obtained.

Thus, setting Γ = 0 in equation (18), we have

$$\frac{d^2T_n(t)}{dt^2}+\alpha_n^2T_n(t)-\frac{P_1^*R_0}{P^*}\sum_{q=1}^{\infty}\frac{d^2T_q(t)}{dt^2}$$
$$+\frac{1}{\mu P^*}\left(F_0P_2^*-N_xk^0-N_yk^1-G_0P_1^*\right)\sum_{q=1}^{\infty}T_q(t)$$
$$=\frac{Mg}{P^*\mu}\Psi_{pi}(ct)\Psi_{pj}(s) \qquad (19)$$

Evidently, an exact analytical solution to this coupled differential equation is not possible. Consequently, the approximate analytical solution technique, which is a modification of the asymptotic method of Struble shall be used (Awodola 2015, Awodola and Oni 2013). Thus, following the procedure in (Awodola and Oni 2013), equation (19) is solved to obtain expression for $T_n(t)$. Thus in view of equation (11), one obtains

$$Z(x,y,t)=\sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty}\frac{K_0\Psi_{pj}(s)}{\gamma_{sf}[\gamma_{sf}^4-\alpha_{pi}^4]}\{[\gamma_{sf}^2 -\alpha_{pi}^2][C_{pi}\gamma_{sf}(\cosh\alpha_{pi}t-\cos\gamma_{sf}t) + B_{pi}(\gamma_{sf}\sinh\alpha_{pi}t-\alpha_{pi}\sin\gamma_{sf}t)] +[\gamma_{sf}^2+\alpha_{pi}^2][A_{pi}\gamma_{sf}(\cos\alpha_{pi}t-\cos\gamma_{sf}t) -(\alpha_{pi}\sin\gamma_{sf}t-\gamma_{sf}\sin\alpha_{pi}t)]\}[\sin\frac{\Omega_{ni}x}{L_X} + A_{ni}\cos\frac{\Omega_{ni}x}{L_X}+B_{ni}\sinh\frac{\Omega_{ni}x}{L_X} + C_{ni}\cosh\frac{\Omega_{ni}x}{L_X}][\sin\frac{\Omega_{nj}y}{L_Y}+A_{nj}\cos\frac{\Omega_{nj}y}{L_Y} + B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y}+C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y}] \quad (20)$$

where

$$\gamma_{sf}=\gamma_s\left[1+\frac{\varepsilon_0P_1^*}{2}\right],\ \gamma_s=\alpha_n+\frac{\lambda P_2^*}{2\alpha_n},$$

$$\lambda=\frac{1}{\mu P^*}\left(F_0-N_x\frac{k^0}{P_2^*}-N_y\frac{k^1}{P_2^*}-G_0\frac{P_1^*}{P_2^*}\right),$$

$$\varepsilon_0=\frac{R_0}{P^*},\ \alpha_{pi}=\frac{\Omega_{pi}c}{L_X},\ K_0=\frac{Mg}{P^*\mu}$$

Equation (20) represents the transverse displacement response to a moving force of a rectangular plate resting on Pasternak elastic foundation.

CASE II:
MOVING MASS PROBLEM

If the mass of the moving load is commensurable with that of the structure, the inertia effect of the moving mass is not negligible. Thus $\Gamma \neq 0$ and one is required to solve the entire equation (18) when no term of the coupled differential equation is neglected. This is termed the moving mass problem. Thus, equation (18) can be rewritten in the form

$$\Bigg[1+\frac{2\varepsilon}{P^*}\left(\frac{P_2^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k) +\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j) +2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\Bigg]\frac{d^2T_n(t)}{dt^2}$$

$$+\frac{4\varepsilon c}{P^*}\left(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k) +\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j) +2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dT_n(t)}{dt}$$

$$+\Bigg[\gamma_{sf}^2+\frac{2\varepsilon c^2}{P^*}\left(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k) +\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j) +2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)\Bigg]T_n(t)$$

$$+\frac{\varepsilon}{P^*}\sum_{\substack{q=1\\q\neq n}}^{}\Bigg[2\left(\frac{P_2^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k) +\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j) +2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right)\frac{d^2T_q(t)}{dt^2}$$

$$4c\left(\frac{P_4^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_4^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_4^{***}(j) +2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_4^{****}(j,k)\right)\frac{dT_q(t)}{dt}$$

$$+2c^2\left(\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j) +2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right)T_q(t)\Bigg]$$

$$=\frac{\varepsilon gL_XL_Y}{P^*}\Psi_{pi}(ct)\Psi_{pj}(s) \quad (21)$$

where $\varepsilon=\dfrac{M}{L_XL_Y\mu}$

Following the procedures in (Awodola and Oni 2013), we have

$$Z(x,y,t)=\sum_{ni=1}^{\infty}\sum_{nj=1}^{\infty}\frac{P_0\Psi_{pj}(s)}{\beta_{sf}[\beta_{sf}^4-\alpha_{pi}^4]}\{[\beta_{sf}^2$$
$$-\alpha_{pi}^2][C_{pi}\beta_{sf}(\cosh\alpha_{pi}t-\cos\beta_{sf}t)$$
$$+B_{pi}(\beta_{sf}\sinh\alpha_{pi}t-\alpha_{pi}\sin\beta_{sf}t)]$$
$$+[\beta_{sf}^2+\alpha_{pi}^2][A_{pi}\beta_{sf}(\cos\alpha_{pi}t-\cos\beta_{sf}t)$$
$$-(\alpha_{pi}\sin\beta_{sf}t-\beta_{sf}\sin\alpha_{pi}t)]\ \}[\sin\frac{\Omega_{ni}x}{L_X}$$
$$+A_{ni}\cos\frac{\Omega_{ni}x}{L_X}+B_{ni}\sinh\frac{\Omega_{ni}x}{L_X}$$
$$+C_{ni}\cosh\frac{\Omega_{ni}x}{L_X}][\sin\frac{\Omega_{nj}y}{L_Y}+A_{nj}\cos\frac{\Omega_{nj}y}{L_Y}$$
$$+B_{nj}\sinh\frac{\Omega_{nj}y}{L_Y}+C_{nj}\cosh\frac{\Omega_{nj}y}{L_Y}] \qquad (22)$$

where

$$\beta_{sf}=\gamma_{sf}\left[1-\frac{\mu_0}{2}\left(R_1-\frac{R_3}{\gamma_{sf}^2}\right)\right],$$

$$\mu_0=\varepsilon\,,\ P_0=\frac{\mu_0 g L_X L_Y}{P^*},$$

$$R_1=\frac{2}{P^*}\left[\frac{P_2^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_3^{**}(k)\right.$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_3^{***}(j)$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_3^{****}(j,k)\right]$$

$$R_3=\frac{2c^2}{P^*}\left[\frac{P_5^*}{2}+\sum_{k=1}^{\infty}\cos\frac{k\pi s}{L_Y}P_5^{**}(k)\right.$$
$$+\sum_{j=1}^{\infty}\cos\frac{j\pi ct}{L_X}P_5^{***}(j)$$
$$\left.+2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\cos\frac{j\pi ct}{L_X}\cos\frac{k\pi s}{L_Y}P_5^{****}(j,k)\right]$$

Equation (22) is the transverse displacement response to a moving mass of a rectangular plate resting on Pasternak elastic foundation. The constants A_{ni}, A_{pi}, A_{nj}, A_{pj}, B_{ni}, B_{pi}, B_{nj}, B_{pj}, C_{ni}, C_{pi}, C_{nj} and C_{pj} are to be determined from the choice of the end support condition.

ANALYSIS OF THE SOLUTION

Examining the phenomenon of resonance; equation (20) clearly shows that the rectangular plate on a uniform Pasternak elastic foundation and traversed by a moving force reaches a state of resonance whenever

$$\gamma_{sf}=\frac{\Omega_{pi}c}{L_X} \qquad (23)$$

while equation (22) shows that the same plate under the action of a moving mass experiences resonance effect whenever

$$\beta_{sf}=\frac{\Omega_{pi}c}{L_X} \qquad (24)$$

where

$$\beta_{sf}=\gamma_{sf}\left[1-\frac{\mu_0}{2}\left(R_1-\frac{R_3}{\gamma_{sf}^2}\right)\right] \qquad (25)$$

Equations (16) and (17) imply that

$$\beta_{sf}=\gamma_{sf}\left[1-\frac{\mu_0}{2}\left(R_1-\frac{R_3}{\gamma_{sf}^2}\right)\right]=\frac{\Omega_{pi}c}{L_X} \qquad (26)$$

Consequently from equations (23) and (26), for the same natural frequency of the plate, the resonance is reached earlier when we consider the moving mass system than when we consider the moving force system.

NUMERICAL CALCULATIONS AND DISCUSSION OF RESULTS

For a cantilever rectangular plate, using the boundary conditions and the initial conditions it can be shown that

$$A_{ni}=\frac{\sinh\Omega_{ni}-\sin\Omega_{ni}}{\cos\Omega_{ni}-\cosh\Omega_{ni}}=\frac{\cos\Omega_{ni}-\cosh\Omega_{ni}}{\sin\Omega_{ni}+\sinh\Omega_{ni}},$$
$$\Rightarrow\quad A_{pi}=\frac{\sinh\Omega_{pi}-\sin\Omega_{pi}}{\cos\Omega_{pi}-\cosh\Omega_{pi}}$$
$$A_{nj}=\frac{\sinh\Omega_{nj}-\sin\Omega_{nj}}{\cos\Omega_{nj}-\cosh\Omega_{nj}}=\frac{\cos\Omega_{nj}-\cosh\Omega_{nj}}{\sin\Omega_{nj}+\sinh\Omega_{nj}},$$
$$\Rightarrow\quad A_{pj}=\frac{\sinh\Omega_{pj}-\sin\Omega_{pj}}{\cos\Omega_{pj}-\cosh\Omega_{pj}} \qquad (27)$$

$$B_{ni}=1,\ \Rightarrow\ B_{pi}=1$$
$$B_{nj}=1\ \Rightarrow\ B_{pj}=1,$$

$$C_{ni}=-A_{ni}\Rightarrow C_{pi}=-A_{pi}$$
$$C_{nj}=A_{nj}\Rightarrow C_{pj}=A_{pj} \qquad (28)$$

and from (27), one obtains

$$\cos\Omega_{ni}\cosh\Omega_{ni} = 1 \quad (29)$$

and

$$\cos\Omega_{nj}\cosh\Omega_{nj} = 1 \quad (30)$$

which are the frequency equations for the dynamical problem, such that

$$\Omega_{1i} = 4.73004,$$
$$\Omega_{2i} = 7.85320,$$
$$\Omega_{3i} = 10.99561 \quad (31)$$

and

$$\Omega_{1j} = 4.73004,$$
$$\Omega_{2j} = 7.85320,$$
$$\Omega_{3j} = 10.99561 \quad (32)$$

Using (27), (28), (31) and (32) in equations (20) and (22) one obtains the displacement response respectively to a moving force and a moving mass of a cantilever (clamped-free) rectangular plate resting on a Pasternak elastic foundation.

For the calculations of practical interests in dynamics of structures, a rectangular plate of length L_Y = 0.914m and breadth L_X = 0.457m is considered. The velocity of the mass is assumed to be 0.8123m/s and the values for E, S and Γ are chosen to be 3.109×10^9kg/m^2, 0.4m and 0.2 respectively.

Figures1 and 2 display the effect of axial forces N_x and N_y respectively on the transverse deflection of the cantilever rectangular plate for the case of moving force. The graphs show that the response amplitudes decrease as the values of N_x and N_y increase.

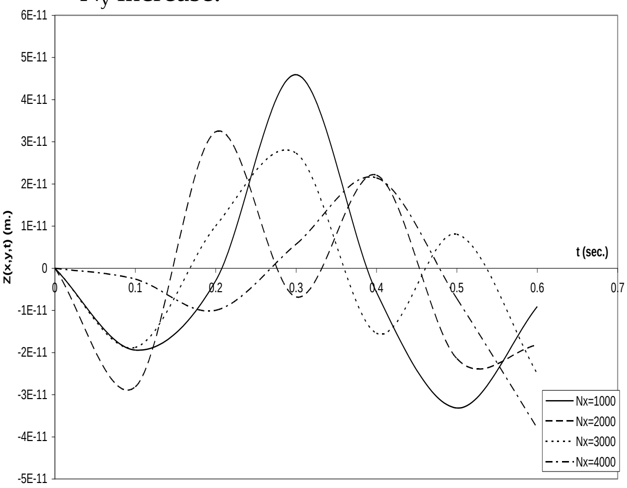

Figure 1: Displacement profile of cantilever plate on Pasternak elastic foundation and traversed by moving force for various values of axial force Nx

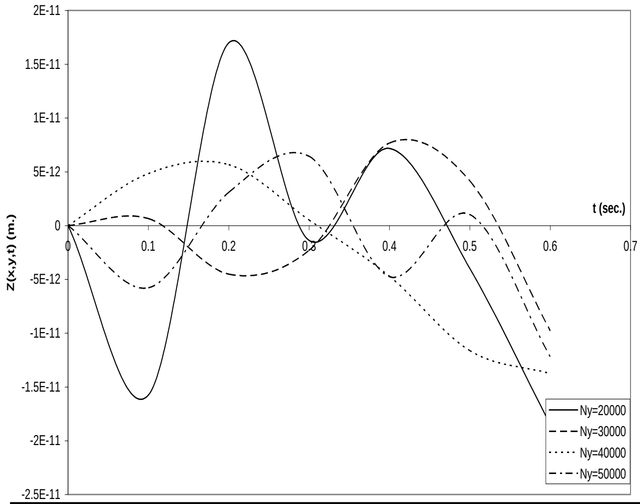

Figure 2: Displacement profile of cantilever plate on Pasternak elastic foundation and traversed by moving force for various values of axial force Ny

The effects of N_x and N_y on the transverse deflection in the case of moving mass displayed in figures 3 and 4 respectively show that an increase in the value of each of N_x and N_y decreases the deflection of the cantilever rectangular plate resting on Pasternak elastic foundation.

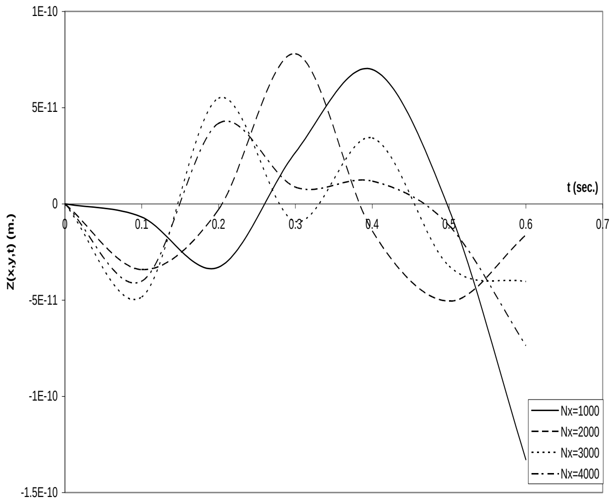

Figure 3: Deflection of cantilever plate on Pasternak Foundation and traversed by moving mass for various values of axial force Nx

Figure 5 compares the displacement curves of the moving force and moving mass for a cantilever rectangular plate for fixed values of N_x and N_y, it is shown that the response amplitude of a moving mass is greater than that of a moving force.

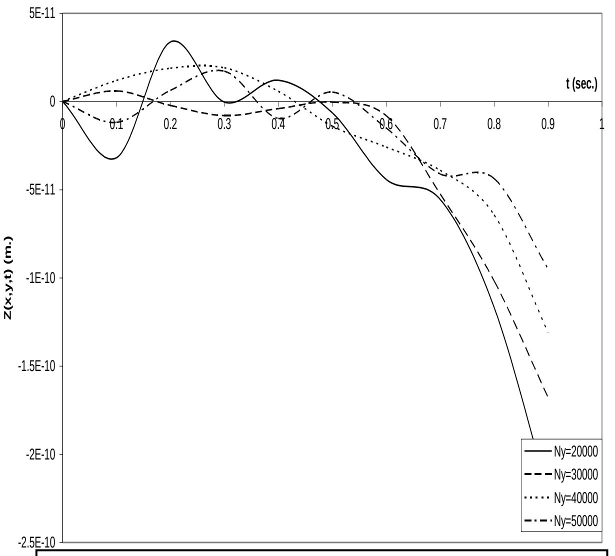

Figure 4: Deflection of cantilever plate on Pasternak Foundation and traversed by moving mass for various values of axial force Ny

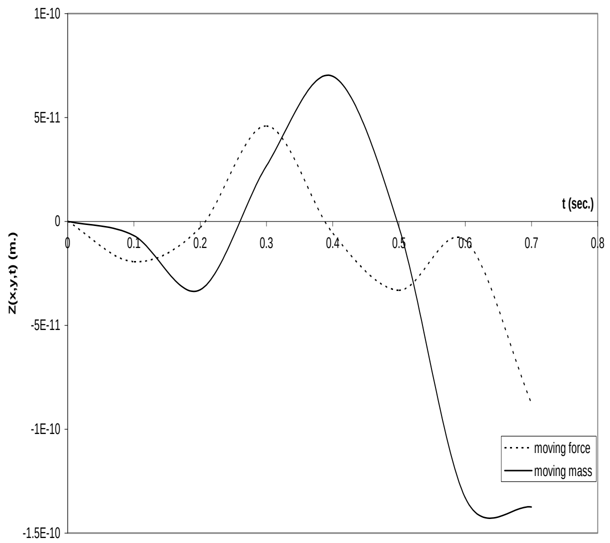

Figure 5: Comparison of moving force and moving mass of cantilever plate resting on Pasternak elastic foundation

CONCLUSION

The influence of prestress on the dynamic response to moving concentrated masses of prestressed cantilever rectangular plates resting on bi-parametric (Pasternak) elastic foundation is considered in this work. The governing non-homogenous fourth order partial differential equation with variable and singular coefficients is first reduced, using a method based on separation of variables, to a set of coupled second order ordinary differential equations. The modified Struble's technique and the method of integral transformations are employed to obtain the closed form solution of the transformed equation for both cases of moving force and moving mass problems of the prestressed cantilever plate under moving load and resting on bi-parametric foundation.

For the same natural frequency of the plate, resonance is reached earlier when we consider the moving mass system than when we consider the moving force system; this implies that the moving force solution is not an upper bound for the accurate solution of the moving mass problem.

It is shown from the results that as the axial forces N_x and N_y increase, the response amplitudes of the cantilever plate decrease for both cases of moving force and moving mass problems, however, higher value of N_y is required for a noticeable effect than N_x, this implies that N_x which is the prestress in the x-direction (the assumed direction of motion) has greater influence on the deflection of the cantilever plate than N_y.
Furthermore, for fixed values of N_x and N_y, the response amplitude for the moving mass problem is greater than that of the moving force problem, this implies that resonance is reached earlier in moving mass problem than in moving force problem, it is therefore unsafe to rely on the moving force solutions.

REFERENCES

Fryba, L. (1972): Vibration of solids and structures under moving loads. Groningen: Noordhoff.

Oni, S. T. (1991): On the dynamic response of elastic structures to moving multi-mass system. Ph.D. thesis. University of Ilorin, Nigeria.

Inglis, C. E. (1934): A mathematical treatise on vibration in railway bridges. The University press, Cambridge.

Gbadeyan, J. A. and Aiyesimi, Y. M. (1990): Response of an elastic beam resting on viscoelastic foundation to a load moving at non-uniform speed. Nigerian Journal of Mathematics and Applications, Vol.3, pp 73-90.

Sadiku, S. and Leipholz, H. H. E. (1981): On the dynamics of elastic systems with moving concentrated masses. Ing. Archiv. 57, 223-242.

Gbadeyan, J. A. and Oni, S. T. (1995): Dynamic behaviour of beams and rectangular plates under moving loads. Journal of Sound and Vibration. 182(5), pp 677-695.

Jaeger,J.C. and Strarfield,A. M. (1979): An introduction to applied Mathematics. Oxford University Press, pp 295-296.

Shadnam M. R., Mofid M. and Akin J. E. (2001): On the dynamic response of rectangular plate with moving mass. Thin-Walled Structures, 39(2001),pp 797 – 806.

Oni S. T. (2004): Flexural motions of a uniform beam under the actions of a concentrated mass traveling with variable velocity. Abacus, Journal of Mathematical Association of Nigeria. Vol. 31, No 2a, pp79–93.

Oni S. T. and Omolofe B. (2005): Dynamic analysis of a prestressed elastic beam with general boundary conditions under moving loads at varying velocities. Journal of Engineering and Engineering Technology,FUTA,Vol. 4,No1, pp55–72.

Oni S. T. and Awodola T. O. (2003): Vibrations under a moving load of a non-uniform Rayleigh beam on variable elastic foundation. Journal of Nigerian Association of Mathematical Physics,Vol.7,pp191–206.

Omer Civalek and Aitung Yauas (2006): Large deflection static analysis of rectangular plates on two parameter elastic foundations. International Journal of Science and Technology. Vol. 1, No. 1, pp 43 – 50.

Adams G. G. (1995): Critical speeds and the response of a tensioned beam on an elastic foundation to repetitive moving loads. Int. Jour. Mech. Sci., Vol. 7, pp 773 – 781.

Savin E. (2001): Dynamics amplification factor and response spectrum for the evaluation of vibrations of beams under successive moving loads. Journal of Sound and Vibrations, 248(2), pp 267 – 288.

Jia-Jang Wu (2006): Vibration analysis of a portal frame under the action of a moving distributed mass using moving mass element. Int. Jour. for Numerical Methods in Engineering, 62, pp 2028 – 2052.

Awodola, T. O. (2014): Mass ratio influence on the response to moving masses of prestressed simply supported plate resting on Winkler foundation. Journal of the Nigerian Association of Mathematical Physics (J. NAMP), 28(2), 81-92.

Clough Ray W. and Penziens, J (1975): Dynamics of structures, Mcgraw-Hill. Inc.

Moshe Eisenberger and Jose Clastornik. (1987): Beams on variable Two-Parameter elastic foundation. Journal of Engineering Mechanics, Vol. 113, No. 10, pp 1454-1466.

Gbadeyan, J. A. and Oni, S. T. (1992): Dynamic response to a moving concentrated masses of elastic plates on a non-Winkler elastic foundation. Journal of Sound and Vibration, 154, pp 343-358.

Awodola, T. O. (2015): Axial force influence on the vibration of prestressed plate with general boundary conditions and resting on Winkler foundation. *FUTA Journal of Research in Sciences,* 11(1), 60-76. (Nigeria, 100%).

Lee H. P. and Ng T. Y. (1996): Transverse vibration of a plate moving over multiple point supports. Applied Acoustics, Vol. 47, No. 4, pp 291 – 301.

Awodola, T. O. and Oni, S. T. **(2013):** Dynamic response to moving masses of rectangular plates with general boundary conditions and resting on variable winkler foundation. Latin American Journal of Solids and Structures (LAJSS), 10(2013), pp 301 – 322.

Oni, S. T. and Awodola T. O. (2010): Dynamic response under a moving load of an elastically supported non-prismatic Bernoulli-Euler beam on variable elastic foundation. Latin American Journal of Solids and Structures. 7, pp 3 - 20.