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SHORT COMMUNICATION

ON THE INFLUENCE OF MASS PER UNIT LENGTH ON THE VIBRATION OF SIMPLY SUPPORTED RAYLEIGH BEAM UNDER THE ACTION OF MOVING LOAD

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ABSTRACT

The dynamic response of a simply supported Rayleigh beam to moving load is investigated in this work. The Finite Fourier Sine transformation is used to reduce the governing fourth order partial differential equation to a second order ordinary differential equation which is solved using the Laplace transformation and the convolution theory. It is observed that as the value of the mass per unit length μ of the beam increases, the deflection amplitude of the beam increases. The investigation of the state of resonance reveals that an increase in the mass per unit length of the beam decreases the critical speed of the moving load. This suggests that additional mass should be reduced as much as possible in order to guarantee the safety of the load moving on the structure [e.g vehicles parking on bridges should be avoided because it adds to the mass of the bridge].

Keywords: Rayleigh beam, Simply supported, Resonance, Rotatory inertial, Critical speed.

INTRODUCTION

Many authors have considered the forced vibrations of elastic bodies such as beams and plates. The force, which causes these vibrations in a beam, may be a function of space co-ordinate only or a force, which varies in both space and time. Among the early researchers in this area of work are Timoshenko (1922), Kenny (1954), Milornir et al (1969)**,** Fryba (1972) and Stanisic et al (1974). Recently, Oni (1990) considered the problem of a harmonic time variable concentrated force moving at a uniform velocity over a finite deep beam. The methods of integral transformations are used. In particular, the finite Fourier transform is used for the length coordinate and the laplace transform for the time coordinate. Series solution, which converges was obtained for the deflection of the simply supported beams. The analysis of the solution was carried out for various speeds of the load. Awodola (2007) considered the variable velocity influence on the vibration of a simply supported Bernoulli-Euler beam, resting on a uniform foundation, under the action of an exponentially varying magnitude load moving with variable velocity. Also, Ouchenane *et al.*, (2009) analyzed the vibration of bridges structures under the influence of moving loads. More recently, Oni and Awodola (2011) investigated the dynamic behaviour under moving concentrated masses of simply supported rectangular plates resting on variable Winkler elastic foundation. It is observed from the aforementioned and many other works in literature that the influence of mass per unit length on the deflection of the beam was not analyzed. Yet, the mass per unit length of the beam is of a great importance in Engineering practice and design. However, this work is set to investigate the influence of mass per unit length on the deflection of the beam. It is also set to analyze the phenomenon of resonance and the relationship between the critical speed and the mass per unit length of the uniform Rayleigh beam.

THE GOVERNING EQUATION

The equation of motion of a uniform Rayleigh beam undergoing transverse vibrations due to moving load is derived using Newton's second law of motion. The equation governing the transverse motion of the uniform Rayleigh beam under the action of a moving load is a fourth order partial differential equation. The governing partial differential equation is given by:

$$\mathrm{EI}\frac{\partial^4 V(x,t)}{\partial x^4} + \mu\frac{\partial^2 V(x,t)}{\partial t^2} - \mu b\frac{\partial^4 V(x,t)}{\partial x^2 \partial t^2} = P(x,t) \quad (1)$$

where

E=Young Modulus

I=Moment of inertia of the cross section

μ =Mass per unit length of the beam

b= Rotatary inertia

V(x, t) = Transverse displacement

P(x, t) = Impressed force

The beam model, taken to be simply supported, has the boundary conditions

$$V(0,t) = V(L,t) = 0 \quad (2)$$

$$\frac{\partial^2 V(0,t)}{\partial x^2} = \frac{\partial^2 V(L,t)}{\partial x^2} = 0 \quad (3)$$

and the initial conditions take the form

$$V(x,0) = 0$$

$$\frac{\partial^2 V(x,0)}{\partial t^2} = 0 \quad (4)$$

The load moving on the elastic beam is assumed a constant magnitude load of the form

$$P(x,t) = P\delta[x-(x_0+\alpha t)] \quad (5)$$

where P indicates the magnitude of the load, α is the speed of the moving load. The function $\delta(x)$ is defined as:

$$\delta(x) = \begin{cases} 0; & if\ x \neq 0 \\ \infty; & if\ x \neq 0 \end{cases} \quad (6)$$

and is called the dirac – delta function with the property:

$$\int_a^b \delta(x-k) f(x) dx = \begin{cases} 0; & for\ k < a < b \\ f(k); & for\ a < k < b \\ 0; & for\ a < b < k \end{cases} \quad (7)$$

Substituting (5) into equation (1), we have

$$\mathrm{EI}\frac{\partial^4 V(x,t)}{\partial x^4} + \mu\frac{\partial^2 V(x,t)}{\partial t^2} - \mu b\frac{\partial^4 V(x,t)}{\partial x^2 \partial t^2} = P\delta[x-(x_0+\alpha t)] \quad (8)$$

METHOD OF SOLUTION

The governing equation (8) is a fourth order partial differential equation with variable co-efficients. To obtain the solution to the differential equation (8), the finite Fourier sine transform is first used to reduce the equation from the fourth order partial differential equation to a second order ordinary differential equation.

The finite Fourier sine transform is given by:

$$V(m,t) = \int_0^L V(x,t) \sin \frac{m\pi x}{L} dx \tag{9}$$

with the inverse

$$V(x,t) = \frac{2}{L} \sum_{m=1}^{\infty} V(m,t) \sin \frac{m\pi x}{L} \tag{10}$$

Applying (9) and (7) in (8), taking into account the boundary conditions (2) and (3), we have

$$EI\left(\frac{m\pi}{L}\right)^4 V(m,t) + \mu \frac{\partial^2}{\partial t^2} V(m,t) + \mu b \frac{\partial^2}{\partial t^2} \left(\frac{m\pi}{L}\right)^2 V(m,t) = \frac{P \sin m\pi (x_0 + \alpha t)}{L} \tag{11}$$

Equation (11) is rearranged to have

$$\frac{d^2 V(m,t)}{dt^2} + R^0 V(m,t) = A^0 \sin\left[\frac{m\pi (x_0 + \alpha t)}{L}\right] \tag{12}$$

where $R^0 = \dfrac{EI\left(\frac{m\pi}{L}\right)^4}{\mu\left[1 + b\left(\frac{m\pi}{L}\right)^2\right]}$ and $A^0 = \dfrac{P}{\mu\left[1 + b\left(\frac{m\pi}{L}\right)^2\right]}$ (13)

In order to solve equation (12), it is simplified to become

$$\frac{d^2 V(m,t)}{dt^2} + R^0 V(m,t) = A^0\left[\sin\left(\frac{m\pi}{L} x_0\right)\cos\left(\frac{m\pi}{L}\alpha t\right) + \cos\left(\frac{m\pi}{L} x_0\right)\sin\left(\frac{m\pi}{L}\alpha t\right)\right] \tag{14}$$

Using the Laplace transform given by

$$L(f(t)) = \int_0^{\infty} e^{-st} f(t) dt$$

Equation (14) is transformed to give

$$V(m,s) = \frac{1}{s^2 + R^0}\left(\frac{H_1 s}{s^2 + c^2} + \frac{H_2 c}{s^2 + c^2}\right) \tag{15}$$

where $H_1 = A^0 \sin \frac{m\pi x_0}{L}$, $H_2 = A^0 \cos \frac{m\pi x_0}{L}$ and $c = \frac{m\pi}{L}\alpha$

In other to find the Laplace inversion of equation (15), we use the convolution theorem defined as

$$L^{-1}[F.G] = f * g \tag{16}$$

where $f * g = \int_0^t f(t-u) g(u) du$

F and G are Laplace transform of f and g respectively

Using (16), the laplace inversion of equation (15) is obtained as

$$V(m,t)=\frac{H1}{R^0}\left[\frac{\sin\sqrt{R^0}t}{2}\left(\frac{\sin\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}+\frac{\sin\left(\sqrt{R^0}-c\right)t}{\sqrt{R^0}-c}\right)-\frac{\cos\sqrt{R^0}t}{2}\left(\frac{\cos\left(\sqrt{R^0}-c\right)t}{c-\sqrt{R^0}}-\frac{\cos\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}-\frac{2\sqrt{R^0}}{c^2-R^0}\right)\right]$$

$$+\frac{H2}{R^0}\left[\frac{\sin\sqrt{R^0}t}{2}\left(\frac{\cos\left(\sqrt{R^0}-c\right)t}{\sqrt{R^0}-c}-\frac{\cos\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}-\frac{2c}{c^2-R^0}\right)-\frac{\cos\sqrt{R^0}t}{2}\left(\frac{\sin\left(\sqrt{R^0}-c\right)t}{\sqrt{R^0}-c}-\frac{\sin\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}\right)\right] \quad (17)$$

Thus, in view of (10), we have that

$$V(x,t)=\frac{2}{L}\sum_{m=1}^{n}\left\{\frac{H1}{R^0}\left[\frac{\sin\sqrt{R^0}t}{2}\left(\frac{\sin\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}+\frac{\sin\left(\sqrt{R^0}-c\right)t}{\sqrt{R^0}-c}\right)\right.\right.$$

$$\left.-\frac{\cos\sqrt{R^0}t}{2}\left(\frac{\cos\left(\sqrt{R^0}-c\right)t}{c-\sqrt{R^0}}-\frac{\cos\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}-\frac{2\sqrt{R^0}}{c^2-R^0}\right)\right]$$

$$+\frac{H2}{R^0}\left[\frac{\sin\sqrt{R^0}t}{2}\left(\frac{\cos\left(\sqrt{R^0}-c\right)t}{\sqrt{R^0}-c}-\frac{\cos\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}-\frac{2c}{c^2-R^0}\right)\right. \quad (18)$$

$$\left.\left.-\frac{\cos\sqrt{R^0}t}{2}\left(\frac{\sin\left(\sqrt{R^0}-c\right)t}{\sqrt{R^0}-c}-\frac{\sin\left(\sqrt{R^0}+c\right)t}{\sqrt{R^0}+c}\right)\right]\right\}\sin\frac{m\pi x}{L}$$

Equation (18) is the response of the simply supported Rayleigh beam under the action of a moving load.

ANALYSIS OF THE ANALYTICAL SOLUTION

In studying undamped system such as this, it is highly desirable to examine the phenomenon of resonance. Equation (18) shows that the simply supported uniform Rayleigh beam traversed by a moving force reaches a state of resonance whenever

$$\sqrt{R^0}=c \quad (19)$$

where $R^0=\dfrac{EI\left(\frac{m\pi}{L}\right)^4}{\mu\left[1+b\left(\frac{m\pi}{L}\right)^2\right]}$ and $c=\dfrac{m\pi}{L}\alpha$

Equation (19) implies that

$$\left(\frac{EI\left(\frac{m\pi}{L}\right)^4}{\mu\left[1+b\left(\frac{m\pi}{L}\right)^2\right]}\right)^{1\backslash 2}=\frac{m\pi}{L}\alpha$$

and hence the critical speed α is gotten to be

$$\alpha = \left(\frac{EI\left(\frac{m\pi}{L}\right)^4}{\mu\left[1 + b\left(\frac{m\pi}{L}\right)^2\right]} \right)^{1\backslash 2} \frac{L}{m\pi} \tag{20}$$

It is observed from equation (20) that if the mass per unit length of the beam increases, the critical speed will reduce. Hence, the mass per unit length of the beam should be made as small as possible to guarantee the safety of the load moving on the structure.

ANALYSIS AND DISCUSSION OF THE RESULT

Here, calculations of practical interests in Dynamics and Engineering design are presented. An elastic beam of length 12.192m has been considered. The flexural rigidity (EI) is taken to be $6.068 \times 10^6 m^3/s^2$ and velocity (α) is taken to be 8.123m/s. The results are displayed in the plotted curves below.

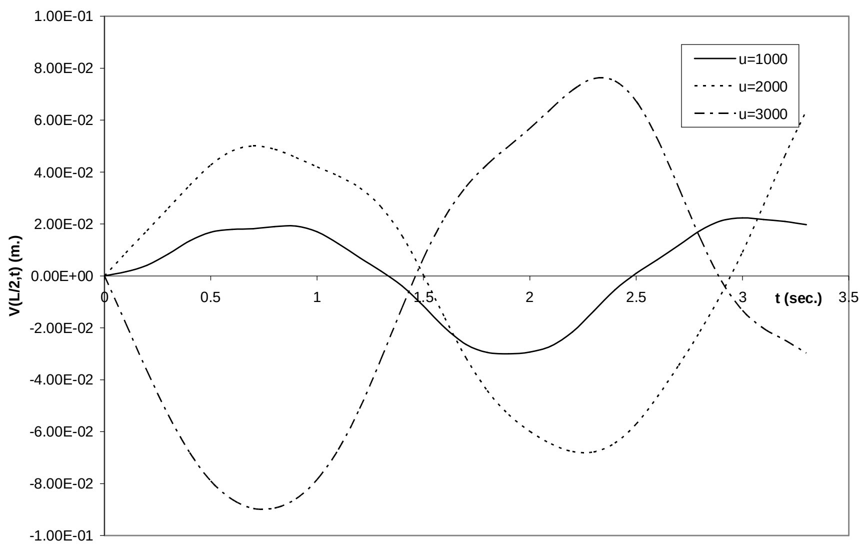

Figure 1: Deflection profile of a uniform Rayleigh beam under moving load for various values of mass per unit length 'µ'.

The figure 1 above shows the displacement response of the simply supported Rayleigh beam under the action of moving load for various values of mass per unit length μ. It is shown that as μ increases the displacement amplitude of the beam increases which implies that an increase in the mass per unit length of the beam reduces the critical speed.

CONCLUSION

The influence of mass per unit length on the transverse deflection of a simply supported Rayleigh beam under the action of moving load has been investigated. The load is assumed to move with constant speed. The governing fourth order partial differential equation is solved and the deflections for various values of the mass per unit length of the beam were obtained and plotted against time (t). The phenomenon of resonance was analyzed. It was found that the deflection amplitude of the beam increases with increase in the value of the mass per unit length. The analysis of resonance reveals that an increase in the mass per unit length of the beam reduces the critical speed of the moving load. This work suggests to the field engineers in the construction of structures such as bridges that reducing the mass per unit length of the structure will guarantee a better safety of the load moving on the structure.

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