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INFLUENCE OF TEMPERATURE DEPENDENT VISCOSITY ON UNSTEADY HYDRO-MAGNETIC FREE CONVECTIVE FLOW ON A POROUS PLATE

A.J. Omowaye

Department of Mathematical Sciences, Federal University of Technology, Akure, Nigeria.

ajomowaye@futa.edu.ng

ABSTRACT

This paper studies the influence of temperature dependent viscosity on unsteady hydro-magnetic free convective flow on a porous plate. The fluid is acted upon by a constant external uniform magnetic field applied perpendicular to the plates. The governing non-linear partial differential equations are solved numerically using finite difference method. Important effect of temperature dependent viscosity and the uniform magnetic field on the flow are discussed. It was shown that there is cross over in the velocity as viscosity parameter increases.

Keyword: Viscosity, free convection, porous plate, hydro-magnetic, Numerical Solution

Nomenclature

English

a Dimensionless Viscosity Parameter.

b Constant associated with Temperature

C_P Specific Heat at Constant Temperature

Ec Eckert Number

g Gravitational Acceleration

G Grashof Number

Ha Hartmman Number

K Thermal Conductivity

P Prandtl Number

$t^{'}$ Time

$T^{'}$ Temperature

$T_W^{'}$ Temperature of the Wall

$T_\infty^{'}$ Temperature at Infinity

$u^{'}$ Fluid Velocity

$v_0^{'}$ normal Velocity of suction / Injection

$y^{'}$ Space Variable

Greek Symbol

β_0 Magnetic Field

$\beta^{'}$ Coefficient of Volume Expansion

ρ Density

σ Electrical Conductivity

υ kinematic Viscosity

μ Dynamic Viscosity

θ Dimensionless Temperature

INTRODUCTION

In recent years, free convection flow of viscous fluids through porous medium have attracted the attention of a number of authors in view of its application to geophysics, astrophysics, meteorology, aerodynamics, boundary layer control and so on.

In addition, convective flow through a porous medium has application in the field of chemical engineering especially in filtration and purification processes. In petroleum technology, to study the movement of natural gas, oil and water through oil channels/reservoir and in the field of agriculture engineering to study the underground resources, the channel flows through porous medium have numerous engineering and geophysical applications (Pathak *et al* (2006).

Hayat *et al*. (2004) investigated the hydro-magnetic oscillatory flow of a fluid bounded by a porous plate when the entire system rotates about axis normal to the plate. And the result showed that the flow fields is appreciably influenced by the material parameter of the third grade fluid, applied magnetic field, the imposed frequency, rotation and suction. Toki and Tokis (2007) showed that unsteady free convection flows on a porous plate with time dependent heating have exact solutions under some initial and boundary condition .these solutions remain bounded in finite time.

In the literature, most of the studies so far are based on constant physical properties of the fluid. More accurate prediction for the flow and heat transfer can be achieved by taking into account the variation of these physical properties, especially the variation of the fluid viscosity with temperature (Herwing and Wicken, 1986). Attia (2006) studied the effect of variable viscosity on the transient couette flow of dusty fluid with heat transfer between parallel plates. The governing non-linear differential equations were solved numerically. It was shown that changing the viscosity variation parameter leads to asymmetric velocity profiles about the central plane of the channel which was similar to effect of variable percolation perpendicular to the plate.

Adesanya *et al*. (2006) modeled a viscous fluid flowing between parallel plates for a temperature dependent viscosity. They investigated the properties of the velocity and they showed that the temperature and velocity fields have two solutions. Ajala *et al*. (2007) presented a reacting system where the thermal conductivity depends linearly on the temperature of the system. The system was solved numerically and they showed that maximum temperature decreases as the thermal conductivity increases. Adesanya and Ayeni (2008) investigated the flow of a reacting pressure/ temperature dependent viscous fluid, when air or oxygen was introduced into the channel containing hydrocarbon ,oxidation or combustion was introduced. The resulting momentum and energy equations were solved numerically and it was shown that the momentum equation has multiple solutions. In an earlier work, Bear (1972) considered the effect of pressure and temperature on viscosity and concluded that most fluids show a pronounced variation with temperature but are relatively insensitive to pressure until high pressures have been attained. He also reported that for gases at twice the critical temperature variations of viscosity with temperature are quite small until pressures of the order of the critical pressure have been reached. However, to illustrate the need to include this viscosity/temperature variation, we quote two examples of common fluids :the viscosity of carbon tetrachloride varies from 1.329 centipoise at 0^oC to 0.384 centipoise at 100^oC; Olive oil has viscosities of 138.0 and 12.4centipoise for respective temperatures of 10^oC and 70^oC Weast (1988-1989). Clearly the variation of viscosity with temperature is an interesting macroscopic phenomenon in fluid mechanics. In spite of its importance in many applications, this effect has received little attention.

Hence, the objective of this paper is to investigate the influence of temperature dependent viscosity on unsteady hydromagnetic free convective flow on a porous plate. The governing non-linear partial differential equations are solved numerically using finite difference method. The influence of temperature dependent viscosity and uniform magnetic field on the flow are discussed.

MATHEMATICAL ANALYSIS.

Consider an unsteady two-dimensional free convective flow, the coordinate origin at an arbitrary point on an infinite, porous limiting vertical plate or wall. The $x^{'}$-axis is along the plate in the upward direction and the $y^{'}$-axis normal towards it. The fluid is viscous and incompressible. The flow is induced either by the motion of the plate or by heating it or by both. The plate is initially at rest and with a constant temperature T_∞ is suddenly moved with the velocity $u_0 p(t^{'})$ in its own plane along the $x^{'}$-axis and its temperature is instantaneously increased (or decreased) by the quantity ($T_w^{'} - T_\infty^{'}$) g(t$^{'}$) for t >0; with u_0 along a constant velocity $T_w^{'}(\neq T_\infty)$ a constant temperature for the plate, p (t$^{'}$) and g(t$^{'}$) two arbitrary functions of non-dimensional time t. An external uniform magnetic field β_0 is applied in the positive $y^{'}$-direction [Attia (2006)]. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected. Also, the viscosity of the fluid is assumed to depend on temperature and is defined as $\mu = \mu_0 f(T)$ for practical reasons which is shown to be suitable for most kinds of fluids (Ames, 1994, Klemp, 1990), the viscosity assumed to vary exponentially with temperature. The function f(T) takes the form f(T) = $e^{-b(T-T_1)}$, where the parameter b has the dimension of $[T]^{-1}$ and such that at T=T_1, f(T_1)=1 and then $\mu = \mu_0$.This means that μ_0 is the viscosity coefficient at T=T_1. A Schematic diagram illustrating the flow domain, the coordinate system and flow parameters is shown in figure 1

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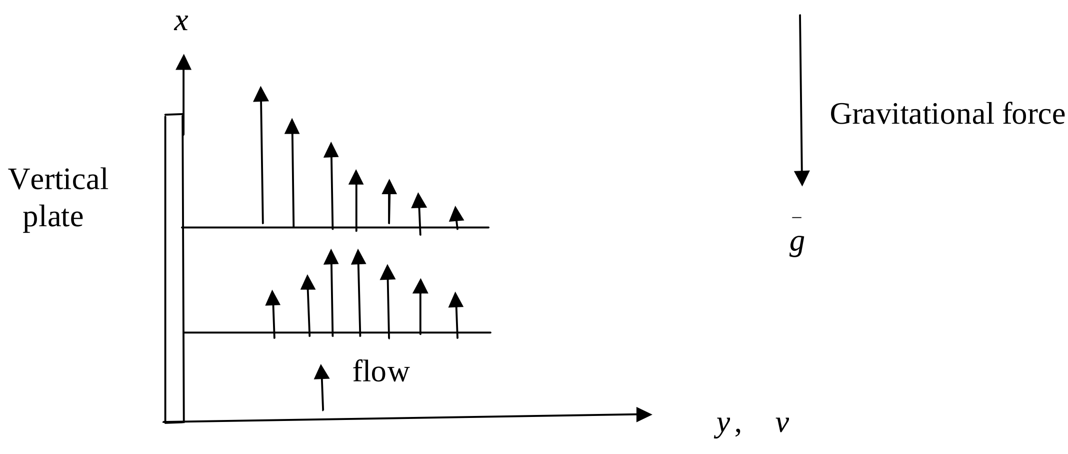

Figure 1: Coordinate system and flow model

Under the above assumptions, the boundary layer equations for unsteady hydromagnetic free convective flow on a porous plate are:

The equation of continuity, on integrating becomes

$$v^{'} = constant = v_0^{'} \quad \text{(say)} \qquad (1)$$

The remaining basic equations of motion and energy for these problems are

$$\frac{\partial u'}{\partial t'} + v_0' \frac{\partial u'}{\partial y'} = \frac{\mu_0}{\rho} \frac{\partial}{\partial y'} \left(e^{-b(T'-T_\infty')} \frac{\partial u'}{\partial y'} \right) + g\beta' (T - T_\infty) - \frac{\sigma \beta_0'^2}{\rho} u', \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v_0' \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{\sigma \beta_0' u'^2}{\rho c_p} . \tag{3}$$

Assuming the no- slip condition occurs between the plate and the fluid the initial and boundary conditions for equations (1)-(3) are

$$u'(y',t') = 0 \quad and \quad T'(y',t') = T_\infty' \quad for \quad y' \geq 0 \quad and \quad t' \leq 0 \tag{4}$$

$$u'(0,t') = [u_0 p(t'),0,0] \quad and \quad T'(0,t') = T_\infty' + (T_w' - T_\infty') g(t') \quad for \quad t' > 0 \tag{5}$$

$$u'(\infty, t'') \to 0 \quad and \quad T'(\infty,t') \to T_\infty \quad for \quad t' \geq 0 \tag{6}$$

Where all symbols are defined in the nomenclature.
We introduce the following non-dimensional variables

$$y = \frac{y' u_0}{\nu}, \; t = \frac{t' u_0^{2'}}{\nu}, \; u = \frac{u'}{u_0}, \; v_0 = \frac{v_0'}{u_0}, \; \theta = \frac{(T' - T_\infty')}{T_W' - T_\infty'} \tag{7}$$

Now substitute (7) into equations (2) – (6) and simplify to have

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = e^{a\theta} \frac{\partial^2 u}{\partial y^2} + a e^{a\theta} \left(\frac{\partial \theta}{\partial y} \right) \left(\frac{\partial u}{\partial y} \right) + Gr\theta - H_a^2 u \tag{8}$$

$$\frac{\partial \theta}{\partial t} + v_0 \frac{\partial \theta}{\partial y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + H_a^2 \, Ec u^2 \tag{9}$$

where the dimensionless viscosity parameter a= $b(T_w - T_\infty)$, the Grashof number Gr=$\frac{g\beta(T_w - T_\infty)\nu}{u_0^3}$, the Hartmann number $H_a^2 = \beta_0 L \sqrt{\frac{\sigma}{\rho \nu}}$,the Prandtl number $\Pr = \frac{\rho c_p \nu}{k}$,the Eckert number $Ec = \frac{u_0^2}{c_p (T_w' - T_\infty')}$.

The boundary conditions are

$$\left.\begin{array}{lll} u(y,0)=0 & \theta(y,0)= 0 & \text{for } y \geq 0, \\ u(0,t) = p(t) & \theta(0,t)= g(t) & \text{for } t>0 , \\ u(\infty,t) \to 0 & \theta(\infty,t) \to 0 & \text{for } t>0 , \end{array}\right\} \tag{10}$$

Using Taylor series $e^{a\theta}$ can be expressed as

$$e^{a\theta} = 1 + a\theta + \frac{(a\theta)^2}{2!} + \frac{(a\theta)3}{3!} + ... \tag{11}$$

We now consider asymptotic expansion of temperature (θ) and velocity (u) in Ha as

$$\left.\begin{array}{l} u = u_0 + H_a u_1 + H_a^2 u_2 + ... \\ \theta = \theta_0 + H_a \theta_1 + H_a^2 \theta_2 + ... \end{array}\right\} \tag{12}$$

$$x = e^{-y} \tag{13}$$

Substituting (11) ,(12), into (8) – (10) and using (13) to transform from infinite domain to finite domain and collecting in order of Ha we have

H_a^0:

$$\frac{\partial u_0}{\partial t} = x^2(1 + a\theta_0 + \frac{(a\theta_0)^2}{2!} + \frac{(a\theta_0)^3}{3!})\frac{\partial^2 u_0}{\partial x^2} + ax^2(1 + a\theta_0 + \frac{(a\theta_0)^2}{2!} + \frac{(a\theta_0)^3}{3!})\left(\frac{\partial \theta_0}{\partial x}\right)\left(\frac{\partial u_0}{\partial x}\right) + v_0 x\frac{\partial u_0}{\partial x} + Gr\theta_0 , \quad (14)$$

$$u_0(x,0) = 0 \qquad u_0(1,t) = p(t) \qquad u_0(0,t) = 0 . \quad (15)$$

H_a^1:

$$\frac{\partial u_1}{\partial t} = x^2(a\theta_1 + \frac{2a^2\theta_0\theta_1}{2!} + \frac{3a^3\theta_0^2\theta_1}{3!})\frac{\partial^2 u_1}{\partial x^2} + v_0 x\frac{\partial u_0}{\partial x} + Gr\theta_0 + ax^2(a\theta_1 + \frac{2a^2\theta_0\theta_1}{2!} + \frac{a^3(3\theta_0^2\theta_2 + 3\theta_0\theta_1^2)}{3!})\left(\frac{\partial \ \theta_1}{\partial x}\right)\left(\frac{\partial \ u_1}{\partial x}\right), \quad (16)$$

$$u_1(x,0) = 0 \qquad u_1(1,t) = 0 \qquad u_1(0,t) = 0 . \quad (17)$$

H_a^2:

$$\frac{\partial u_2}{\partial t} = x^2(a\theta_2 + \frac{a^2(2\theta_0\theta_2 + \theta_1^2)}{2!} + \frac{a^3(3\theta_0^2\theta_2 + 3\theta_0\theta_1^2)}{3!})\frac{\partial^2 u_2}{\partial x^2} + v_0 x\frac{\partial u_2}{\partial x} + Gr\theta_0 + ax^2(a\theta_2 + \frac{a^2(2\theta_0\theta_2^2 + \theta_1^2)}{2!} + \frac{a^3(3\theta_0^2\theta_2 + 3\theta_0\theta_1^2)}{3!})\left(\frac{\partial \ \theta_2}{\partial x}\right)\left(\frac{\partial \ u_2}{\partial x}\right) - u_0 , \quad (18)$$

$$u_2(x,0) = 0 \qquad u_2(1,t) = 0 \qquad u_2(0,t) = 0 . \quad (19)$$

H_a^0:

$$pr\frac{\partial \theta_0}{\partial t} = x^2\frac{\partial^2 \theta_0}{\partial x^2} + Prx\frac{\partial \theta_0}{\partial x} , \quad (20)$$

$$\theta_0(y,0) = 0 \qquad \theta_0(1,t) = g(t) \qquad \theta_0(0,t) = 0 . \quad (21)$$

H_a^1:

$$pr\frac{\partial \theta_1}{\partial t} = x^2\frac{\partial^2 \theta_1}{\partial x^2} + Prx\frac{\partial \theta_1}{\partial x} , \quad (22)$$

$$\theta_1(y,0) = 0 \qquad \theta_1(1,t) = 0 \qquad \theta_1(0,t) = 0 . \quad (23)$$

H_a^2:

$$pr\frac{\partial \theta_2}{\partial t} = x^2\frac{\partial^2 \theta_2}{\partial x^2} + Prx\frac{\partial \theta_2}{\partial x} + \Pr Ecu_0^2 , \quad (24)$$

$$\theta_2(y,0) = 0 \qquad \theta_2(1,t) = 0 \qquad \theta_2(0,t) = 0 . \quad (25)$$

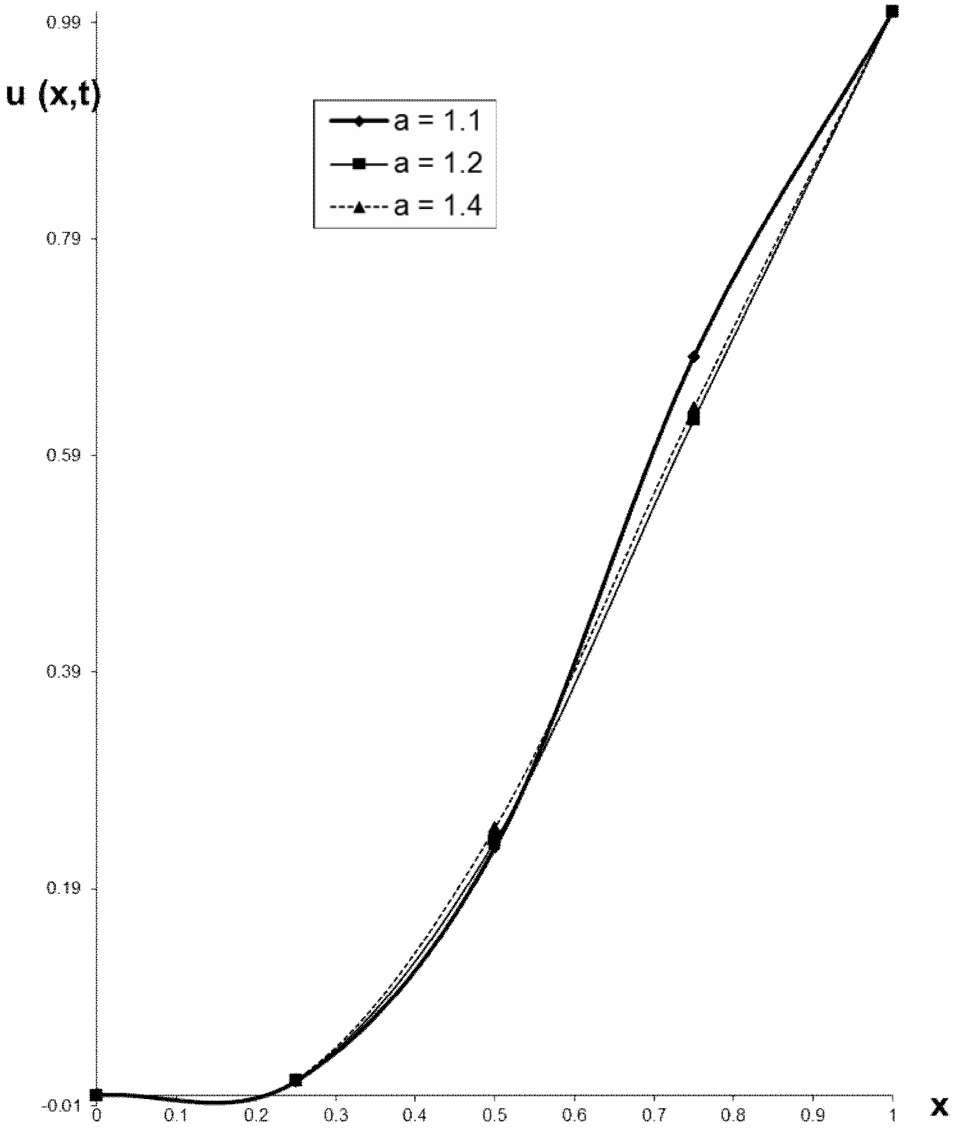

Figure 2:The graph of unsteady velocity distribution u (x,t) against x of a temperature dependent flow for equation 14 when G = 1.0,P= 1.0,Vo = -1.0,Ec = 1.0,t = 0.2 for various a

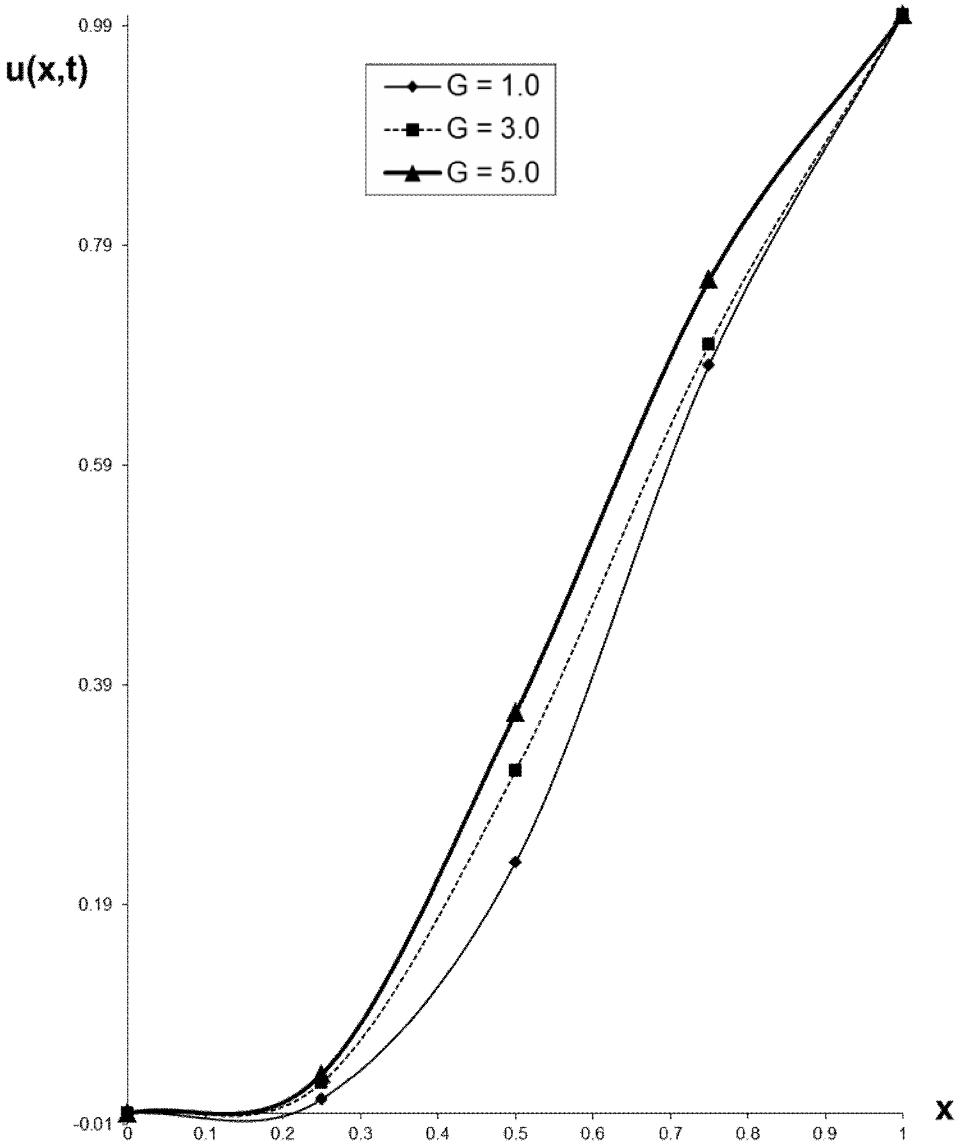

Figure 3: The graph of unsteady velocity distribution u(x,t) against x of a temperature dependent flow for equation 14 when P = 1.0, Vo = -1.0, Ec = 1.0, a = 1.1, t = 0.2 for various G

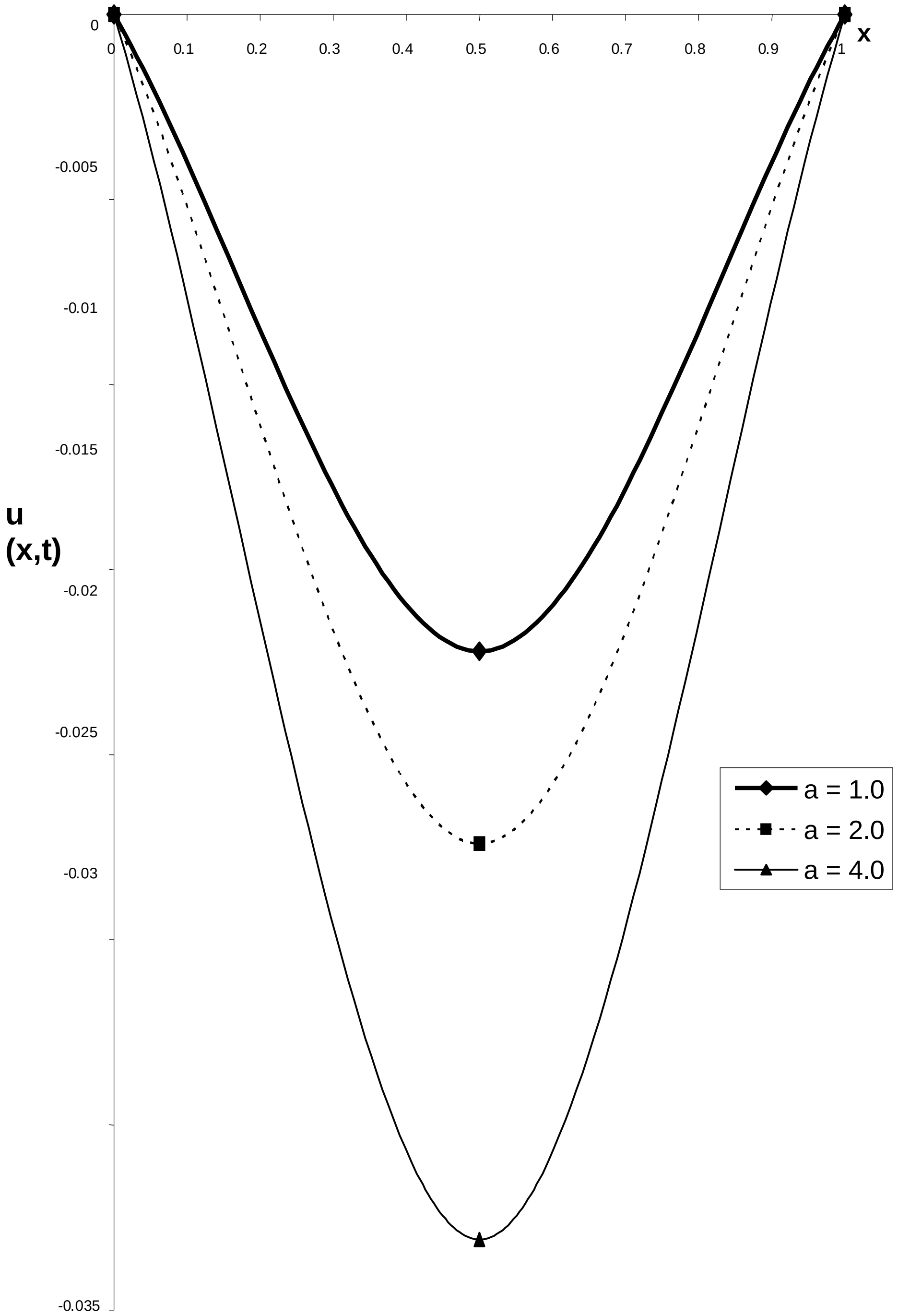

Figure 4:The graph of unsteady velocity distribution u (x,t) against x of a temperature dependent flow for equation 18 when P = 1.0,Vo = -1.0,Ec = 1.0,t = 0.2 for various a

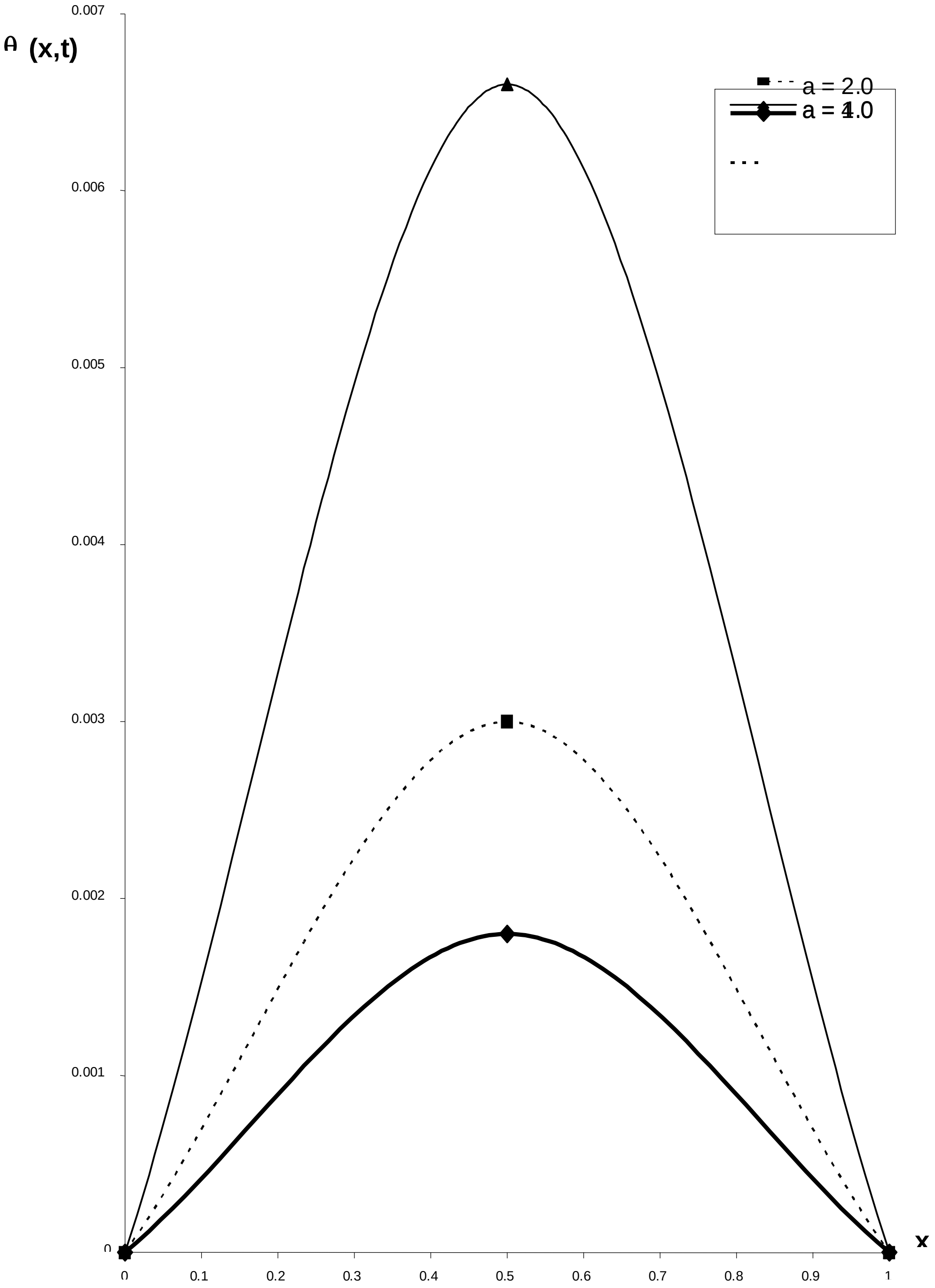

Figure 5:The graph of unsteady temperature distribution θ (x,t) against x of a temperature dependent flow for equation 24 when P = 1.0,Vo = -1.0,Ec = 1.0,t = 0.2 for various a

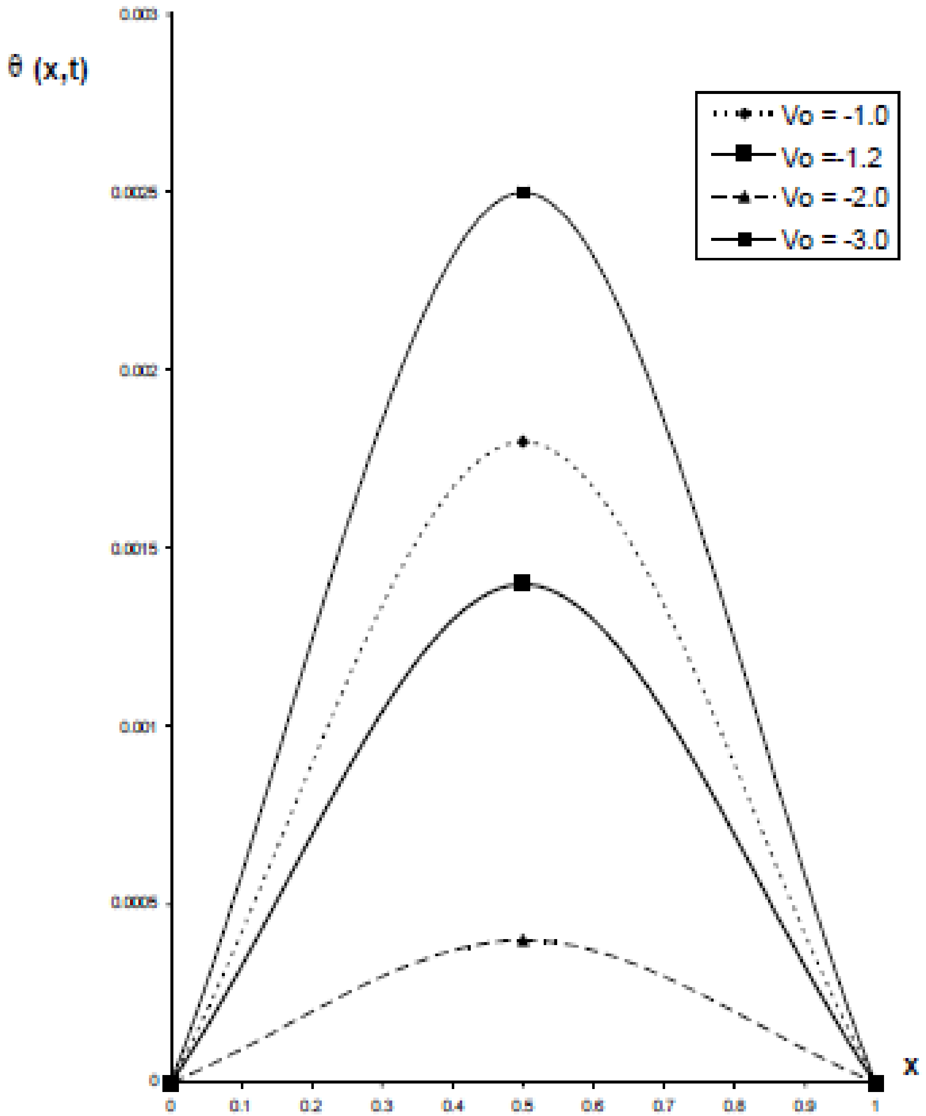

Figure 6:The graph of unsteady temperature distribution θ (x,t) against x of a temperature dependent flow for equation 24 when P= 1.0,Ec = 1.0,a =1.0,t =0.2 for various Vo

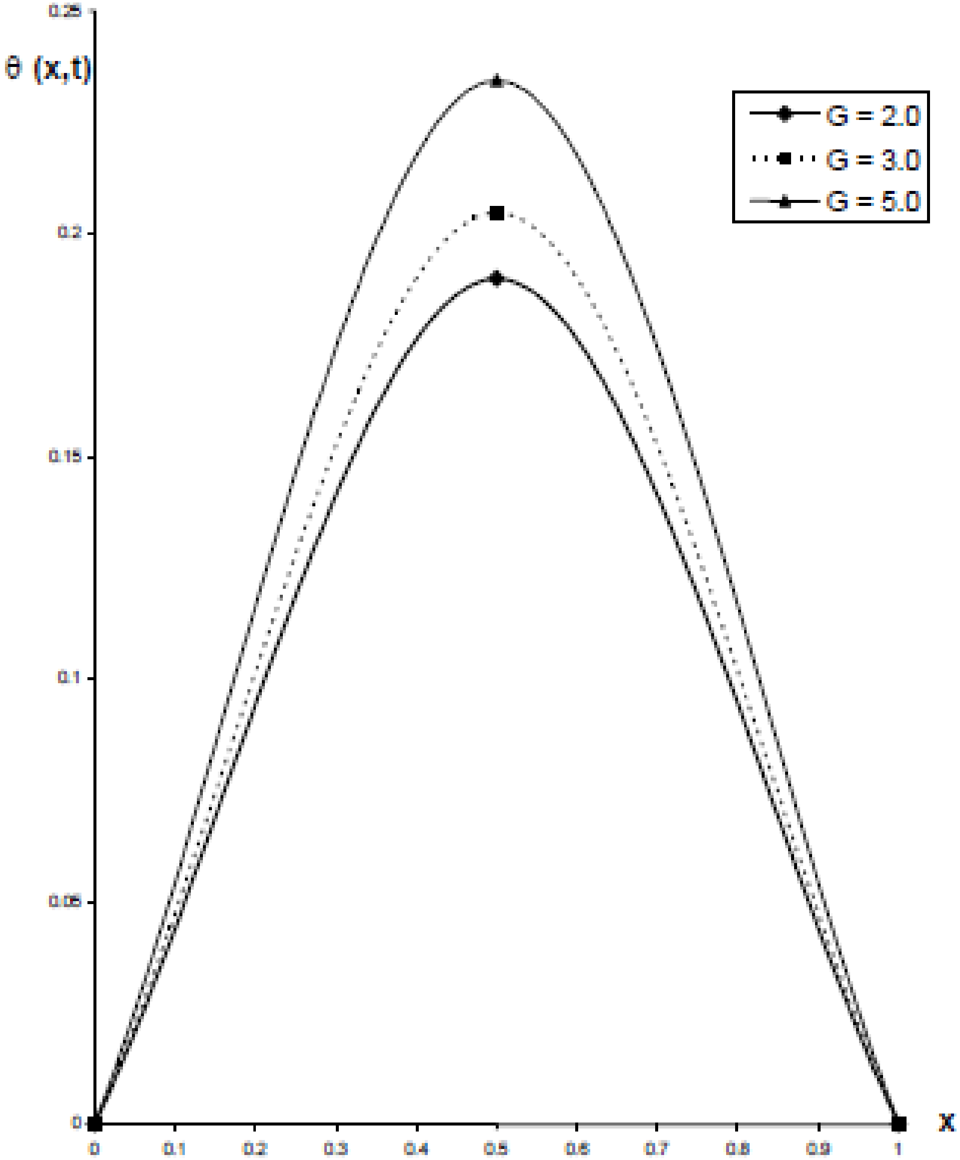

Figure 7: The graph of unsteady temperature distribution $\theta(x,t)$ against x of a temperature dependent flow for equation 24 when P =1.0, Vo = -1.0, Ec =1.0, a = 1.0, t = 0.2 for various G

Assume for this analysis p(t)=g(t)=1.The system of coupled and non-linear partial differential equations 14,16,18,20,22,24 along with the boundary conditions 15,17,19,21,23,25 have been solved numerically by an implicit finite difference scheme of Crank-Nicolson type .The essential features of this technique is that it is based on a finite difference scheme, which has better stability, simple, accurate and more efficient. This technique leads to a system which is tri-diagonal and therefore speedy convergence as well as economical memory space to store coefficient for detail (Turner,1994; Tyn,Mymt and Lakenath, 1987).

DISCUSSION OF RESULTS

In this section, the main results of this study are presented in the form of non-dimensional velocity and temperature profiles. Six basic parameters governed the flow, they are Prandtl number (Pr), suction velocity (v_0), Hartmann number (Ha), Grashof number(Gr), Eckert number (Ec) and non-dimensional viscosity parameter (a). We must remark here that the exponential dependence of the viscosity on temperature results in decomposing the viscous force term in the momentum equation into two terms. The variation of these resulting terms with the viscosity parameter (a) and their relative magnitudes have effect on the flow and temperature field in the absence or presence of the applied uniform magnetic field. Figures 1-3 show the variation of velocities for different values of viscosity parameter, suction parameter, Grashof number on order zero velocity, it is observed that, the velocity increases as the viscosity parameter, suction velocity and Grashof number increase. Also, there is cross over of velocity as viscosity parameter increases as shown in figure1. Figure 4 illustrates the effect of viscosity parameter on order second velocity profile as viscosity parameter increases the velocity decreases. Figures 5,6,7 illustrate the effect of viscosity parameter, suction velocity and Grashof number on order seconded temperature profile . It is interesting to observe that the temperature increases gradually from the plate and reaches peak within the fluids and gradually decrease as it moves away from the plate. In all, the temperature increase as viscosity parameter, suction velocity and Grashof number increase. Also, the order first velocity and temperature profiles has zero as its value.

Finally the influence of the magnetic field (Hartmann number) is what we observed in figure 4 as we know the introduction of magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem.

CONCLUSION

In this paper, the influence of temperature dependent viscosity on unsteady hydro-magnetic free convective flow on a porous plate is considered, the viscosity was assumed to vary exponentially with temperature. The most interesting result was the crossover of the velocity curves due to the variation of the parameter a.

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